

Heterogeneous Agent Trade

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July 8, 2024

The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. This project was developed with research support from the National Science Foundation (NSF Award number 1948800). Thomas Hasenzagl and Teerat Wongrattanapiboon provided excellent research assistance.

Heterogenous Price Elasticities and Trade

Two ingredients:

- Trade as in Armington, but households have random utility over varieties — [McFadden \(1974\)](#).
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks — [Bewley \(1979\)](#).

Two core results:

- A household's price elasticity, in essence, is about the marginal gain in utility from a percent change in consumption \Rightarrow the poor are the most price sensitive.
- The gains from trade — to a first order — depend on these elasticities. And can \Rightarrow pro-poor gains from trade.

Quantitatively, these forces are powerful — the poorest households gain 4.5X more than the richest; the average gains from trade are 3X than representative agent benchmarks.

Outline

1. Illustrative model

- Static framework — intended to illustrate how everything works.

2. Main model

- Dynamic framework — production side + the standard incomplete markets model.

3. Quantitative results

- Calibrated to bilateral trade flows and micro evidence. Gains from trade calculations.

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Illustrative Model — Environment

Armington model with two countries home and foreign.

Continuum of agents with names $k \in [0, 1]$ in the home country. Agents are heterogeneous in wealth w^k . Agents have preferences:

$$u(c_{hh}^k) + \epsilon_h^k \quad \text{and} \quad u(c_{hf}^k) + \epsilon_f^k,$$

- discrete choice... so each household chooses only the home or foreign variety.
- ϵ_j^k s are iid across agents and distributed Type 1 Extreme Value with parameter σ_ϵ .
- For now, u is well behaved.

Agents **maximize expected utility** by formulating state contingent plans of variety choice based upon all possible realizations of ϵ_h, ϵ_f .

Illustrative Model — Trade

Micro-level trade

$$\pi_{hf}^k = \exp\left(\frac{u(w^k/p_{hf})}{\sigma_\epsilon}\right) / \Phi_h^k, \quad \text{where } \Phi_h^k := \sum_{j \in h, f} \exp\left(\frac{u(w^k/p_{hj})}{\sigma_\epsilon}\right),$$

which is the standard result with the Type 1 EV taste shocks.

Aggregate trade arises from the **explicit aggregation** of these decisions

$$M_{hf} = \int w^k \pi_{hf}^k dk.$$

The **aggregate trade elasticity** is just a expenditure weighted average of all agent k s elasticities.

- Micro-elasticities. . . next slide

Agent k 's trade elasticity

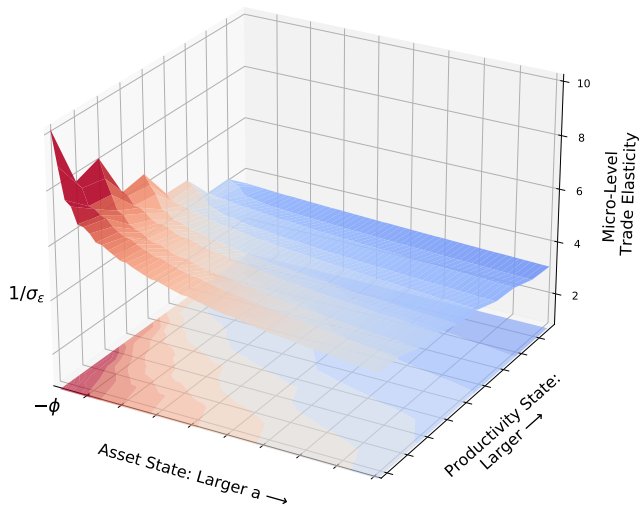
$$-\theta_{hf,f}^k = \frac{1}{\sigma_\epsilon} \times \left[u'(c_{hf}^k) c_{hf}^k \right]$$

Lesson #1: This model easily leads to heterogeneous price sensitivity and specifically the idea that **the poor are the most price sensitive**.

- Generally depends on how curved u is ... with CRRA and curvature parameter $> 1 \Rightarrow$ elasticities are larger for the poor.
- Intuition: A price reduction delivers a lot of extra utility for high marginal utility (poor) households and this induces strong substitution by the poor.

A second interesting case is when u is log ... elasticities are constant and this leads to the [Anderson et al. \(1987\)](#) CES aggregation result.

Example: Micro-Level Trade Elasticities by Income and Wealth



What are the gains from a reduction in the price of the foreign good?

In equivalent variation units, the first order gains from trade are . . .

$$EV^k \approx -\pi_{hf}^k \times \left[\frac{\theta_{hf,f}^k}{\sum_j \pi_{hj}^k \times \theta_{hj,j}^k} \right] \times \Delta \log p_{hf}$$

The gains are about (i) exposure ([Deaton \(1989\)](#), [Borusyak and Jaravel \(2021\)](#)) and (ii) elasticities ([Auer, Burstein, Lein, and Vogel \(2022\)](#)).

Lesson #2: In this model, elasticities matter for the gains from trade and how they are distributed.

- The standard result is that elasticities don't matter to a first order.
- **Here they do.** And under certain conditions, these gains are pro-poor.
- Why? Next slide. . .

Why do elasticities matter?

The same equivalent variation formula can be rewritten as

$$EV^k \approx -\pi_{hf}^k \times \left[\frac{u'(c_{hf}^k)/p_{hf}}{\sum_j \pi_{hj}^k \times u'(c_{hj}^k)/p_{hj}} \right] \times \Delta \log p_{hf}$$

Lesson #3: Elasticities matter because **markets are incomplete**. Agents lack markets to insure against the desire to consume a good. With complete markets one has

$$u'(c_{hj}^k)/p_{hj} = u'(c_{hj'}^k)/p_{hj'} \quad \forall j, j'$$

and the term in brackets disappears.

Intuition: If the price reduction occurs on the high price good, this helps equate marginal utility across goods \Rightarrow an additional, first order gain from trade.

And the gains can be pro-poor when the poor are more “misallocated.”

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Connections and implications:

- **Theory:** Relates to the spatial analysis of [Donald, Fuku, and Miyauchi \(2024\)](#) and complete markets decentralization of discrete choice models in [Mongey and Waugh \(2023\)](#).
- **Measurement:** Assumptions when micro-level shares (the π^k s) are constructed?

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Motivating the Main Model

Two enrichments:

1. Production side to connect prices with trade flows as in quantitative trade models.
2. The standard incomplete markets model on the household side. This is important because...
 - In the illustrative model, there was no ability to insure. The SIM model provides an **intermediate setting** with self-insurance, and thus works against the “elasticity effects”.
 - Less important reasons (i) I like it (ii) I started there (iii) provides an interface with large body of research in macro (iv) a theory as to why there are rich and poor people.

Main Model: Overview

M countries. Each country produces a nationally differentiated product as in Armington; competitive firms with linear technologies in labor and trade costs d_{ij} .

Continuum of households $k \in [0, L_i]$ in each country i . Household preferences:

$$E \sum_{t=0}^{\infty} \beta^t \tilde{u}_{ijt}^k,$$

where conditional direct utility for good j is

$$\tilde{u}_{ijt}^k = u(c_{ijt}^k) + \epsilon_{jt}^k, \quad j = 1, \dots, M.$$

- discrete-continuous choice. . . so first chose one variety, then continuous choice over quantity.
- ϵ_{jt}^k s are iid across hh and time; distributed Type 1 Extreme Value with parameter σ_{ϵ} .

Household k 's efficiency units z_t evolve according to a Markov Chain, and can borrow or accumulate a non-state contingent asset, a , with gross return R_i .

The HA Trade Elasticity

How do imports change relative to domestic consumption due to a permanent change in d_{ij} ?

Proposition 1 (The HA Trade Elasticity)

The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_{z,a} \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) da dz - \int_{z,a} \left\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \right\} \omega_{ii}(a,z) da dz$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \quad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^I = \left[- \frac{\partial a'_i(a,z,j)/p_{ij} c_i(a,z,j)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.$$

The idea: a lower d_{ij} relaxes the bc and then the division of new resources between assets and expenditure determines the intensive margin. **This is larger for the poor, smaller for the rich.**

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = \frac{1}{\sigma_\epsilon} \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}.$$

Now assume the number of countries is large...

The HA Trade Elasticity

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which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a, z)^I = \frac{\partial c_i(a, z, j) / c_i(a, z, j)}{\partial d_{ij} / d_{ij}}, \quad \theta_{ij}(a, z)^E = \frac{\partial \pi_{ij}(a, z) / \pi_{ij}(a, z)}{\partial d_{ij} / d_{ij}},$$

and $\omega_{ij}(a, z)$ are the expenditure weights.

$$\theta_{ij}(a, z)^E \approx -\frac{1}{\sigma_\epsilon} \left[u'(c_i(a, z, j)) c_i(a, z, j) \right].$$

Exactly the same as in the illustrative model. With CRRA and relative risk aversion > 1 then **poor hh's are the most price sensitive on the extensive margin.**

HA Gains from Trade

How does utility change under the heuristic of an immediate jump to the new steady state?

Proposition 2 (HA Gains from Trade)

Household level gains are given by

$$\frac{dv_i(a, z)}{dd_{ij}/d_{ij}} = \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \left\{ A(a_t, z_t) + B(a_t, z_t) + C(a_t, z_t) \right\}.$$

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This term is what I call the “gains from substitution”:

$$A(a, z) = -\sigma_\epsilon \frac{d\pi_{ij}(a, z)/\pi_{ij}(a, z)}{dd_{ij}/d_{ij}}$$
$$\approx \sigma_\epsilon \times \pi_{ij}(a, z) \times \bar{\theta}(a, z)_{ij,j}^E$$

Same idea as in illustrative model — the last line says these gains are about (i) exposure and (ii) elasticities.

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This term is what I call “valuation effects”:

$$B(a, z) = u'(c_i(a, z, i)) \times a \times \frac{dR_i/w_i}{dd_{ij}/d_{ij}}$$

How hh's real wealth (+ or -) change through GE effects on prices — all evaluated at the hh's marginal utility of home consumption.

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This term is what I call the “gains from changes in asset holdings”

$$C(a, z) = \underbrace{\left\{ -\frac{u'(c_i(a, z, i))}{p_{ii}} + \beta \mathbb{E}_{z'} \left[-\sigma_{\epsilon} \frac{\partial \pi_{ii}(a', z') / \pi_{ii}(a', z')}{\partial a'} + \frac{u'(c_i(a', z', i)) R_i}{p_{ii}} \right] \right\}}_{\text{Euler equation}} \frac{dg_i(a', z', i)}{dd_{ij}/d_{ij}}$$

which is zero for small changes as hh's are either (i) on their Euler equation or (ii) constrained and can't adjust their asset position.

Proposition 3 (Separation of Trade and Micro-Heterogeneity)

In the heterogeneous agent trade model where preferences are logarithmic over the physical commodity, the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and trade flows satisfy a standard gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i} \right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and both are independent of the household heterogeneity. And the welfare gains from trade for an individual household are

$$\frac{dv_i(a, z)}{dd_{ij}/d_{ij}} = \frac{1}{\theta(1-\beta)} \times \frac{d\pi_{ij}/\pi_{ij}}{dd_{ij}/d_{ij}} + \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \left\{ B(a_t, z_t) + C(a_t, z_t) \right\}.$$

This mimics the results of [Anderson, De Palma, and Thisse \(1987\)](#). This was not obvious to me given the environment . . . risk, market incompleteness, borrowing constraints, etc.

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And we are back to [Arkolakis et al. \(2012\)](#) + what's going on with factor prices and borrowing constraints.

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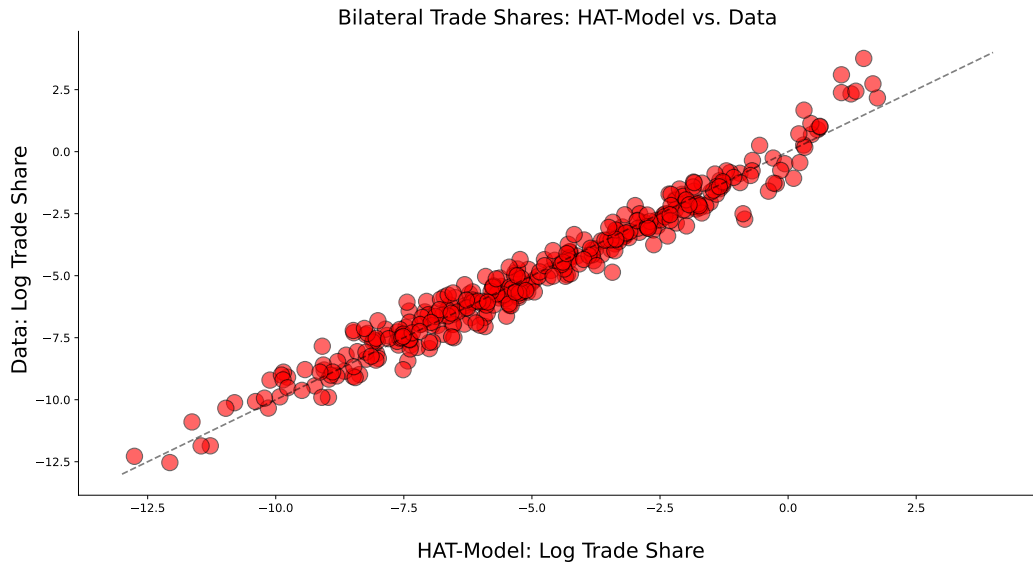
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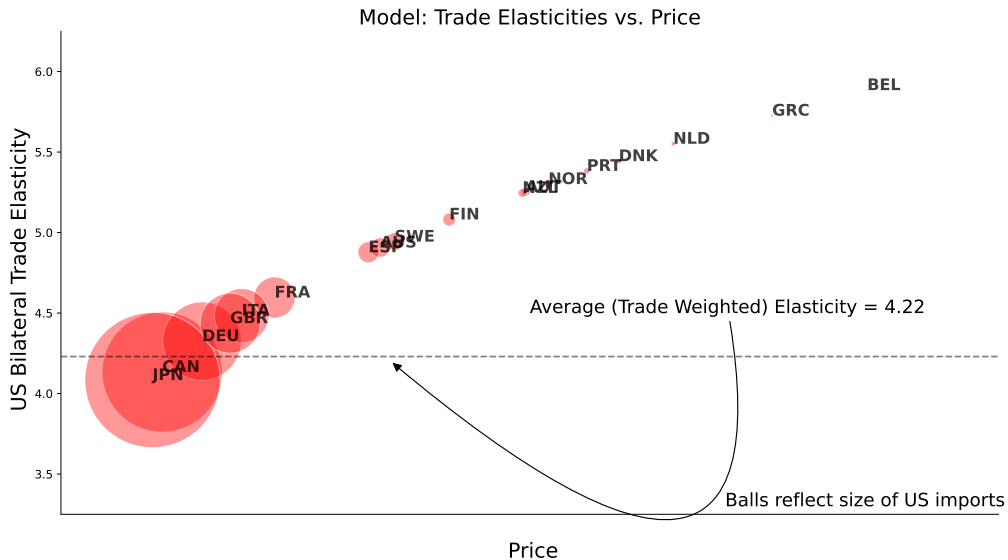
Quantitative Analysis

This is what I'll do...

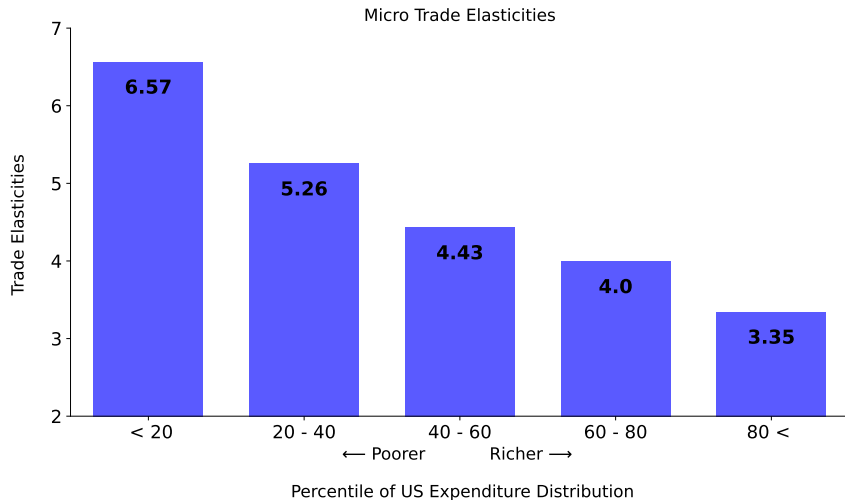
1. Calibrate my model using my “gravity as a guide” approach on the 19 country data set of [Eaton and Kortum \(2002\)](#) and targeting micro-evidence from [Borusyak and Jaravel \(2021\)](#) and [Auer et al. \(2022\)](#).
2. Gains from trade calculations.



US Trade Elasticities: $-\theta_{us,j}$

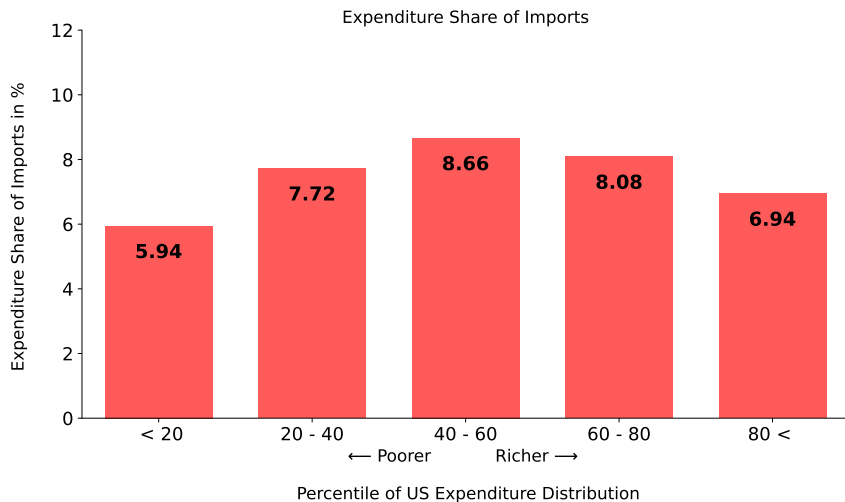


Micro Moments — Model Consistent with HH-Level Elasticities

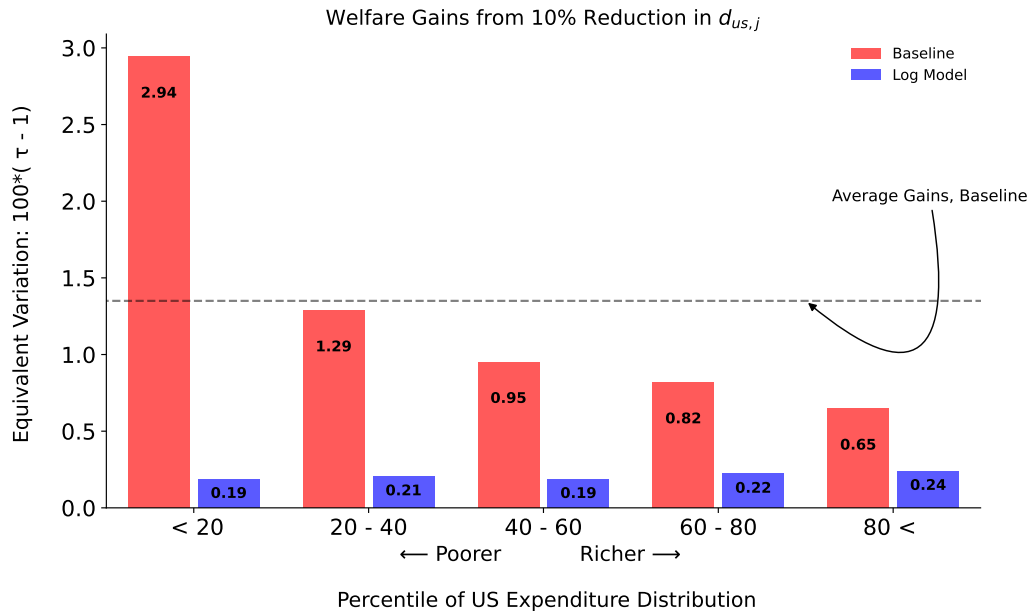


- Household-level elasticities consistent with those in [Auer, Burstein, Lein, and Vogel \(2022\)](#), i.e. rich less elastic than the poor.

Micro Moments — Model Consistent with HH-Level Expenditure Patterns



- Household-level import shares consistent with facts from [Borusyak and Jaravel \(2021\)](#), i.e. rich and poor do not spend unequally on imports.



Final Thoughts...

This paper has prompted even more questions...

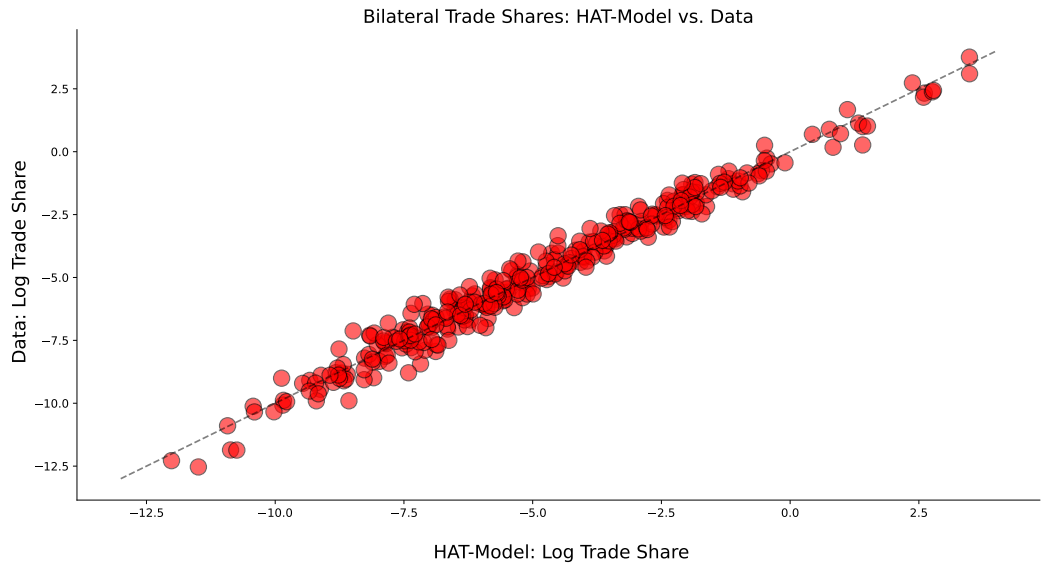
- The efficient pattern of trade? In a companion paper, I show that near-shoring is an outcome that a global planner likes.
- Can trade policy improve outcomes? Put in tariffs and redistribute.
- The interaction between trade goods and trade in assets?

One more thing: My github repository provides the code and supplementary work behind this paper at <https://github.com/mwaugh0328/heterogeneous-agent-trade>.

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Log Model — Fit of Trade Data



Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i , competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i .

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Continuum of households $k \in [0, L_i]$ in each country i . Household preferences:

$$E \sum_{t=0}^{\infty} \beta^t \tilde{u}_{ijt}^k,$$

where conditional direct utility for good j is

$$\tilde{u}_{ijt}^k = u(c_{ijt}^k) + \epsilon_{jt}^k, \quad j = 1, \dots, M.$$

Assumptions:

- discrete-continuous choice. . . so first chose one variety, then continuous choice over quantity.
- ϵ_{jt}^k s are iid across hh and time; distributed Type 1 Extreme Value with dispersion parameter σ_ϵ .
- For now, u is well behaved.

Model: Households II

Household k 's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it} .

Households borrow or accumulate a non-state contingent asset, a , with gross return R_i . Household's face the debt limit

$$a_{t+1}^k \geq -\phi_i.$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij} c_{ijt}^k + a_{t+1}^k \leq R_i a_t^k + w_{it} z_t^k.$$

What Households Do I

Focus on a stationary setting. A hh's state are its asset holdings a and shock z .

1. Condition on variety choice their problem is:

$$v_i(a, z, j) = \max_{a', c_{ij}} \left\{ u(c_{ij}) + \beta \mathbb{E}[v_i(a', z')] \right\},$$

$$\text{subject to } p_{ij}c_{ij} + a' \leq R_i a + w_i z \quad \text{and} \quad a' \geq -\phi_i.$$

2. The ex-post value function of a household in country i is

$$\max_j \{ v_i(a, z, j) + \epsilon_j \}.$$

What Households Do II

Three equations characterizing the commodity choice, value functions, consumption / savings...

1. The choice probability is:

$$\pi_{ij}(a, z) = \exp\left(\frac{v_i(a, z, j)}{\sigma_\epsilon}\right) / \Phi_i(a, z),$$

$$\text{where } \Phi_i(a, z) := \sum_{j'} \exp\left(\frac{v_i(a, z, j')}{\sigma_\epsilon}\right).$$

2. The ex-ante value function of a household in country i is

$$v_i(a, z) = \sigma_\epsilon \log \{\Phi_i(a, z)\}.$$

3. Away from the constraint, consumption and asset choices must respect this Euler equation:

$$\frac{u'(c_i(a, z, j))}{p_{ij}} = \beta R_i E_{z'} \left[\sum_{j'} \pi_{ij'}(a', z') \frac{u'(c_i(a', z', j'))}{p_{ij'}} \right].$$

Definition 1 (The Decentralized Stationary Equilibrium)

A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{ g_i(a, z, j), \pi_{ij}(a, z) \}_i$, probability distributions $\{ \lambda_i(a, z) \}_i$ and positive real numbers $\{ w_i, p_{ij}, R_i \}_{i,j}$ such that

- i Prices (w_i, p_{ij}) satisfy firms problem;
- ii The policy functions and choice probabilities solve the household's problem;
- iii The probability distribution $\lambda_i(a, z)$ induced by the policy functions, choice probabilities, and primitives satisfies the law of motion and is stationary;
- iv Goods market clears:

$$p_i Y_i - \sum_j X_{ji} = 0, \quad \forall i$$

- v Bond market clears with either

$$A'_i = 0, \quad \forall i \quad \text{or} \quad \sum_i A'_i = 0$$

The HA Trade Elasticity

How do i 's imports from j change relative to domestic consumption due to a permanent change in d_{ij} ?

Proposition 4 (The HA Trade Elasticity)

The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_{z,a} \left\{ \theta_{ij}(a, z)^I + \theta_{ij}(a, z)^E \right\} \omega_{ij}(a, z) da dz - \int_{z,a} \left\{ \theta_{ii,j}(a, z)^I + \theta_{ii,j}(a, z)^E \right\} \omega_{ii}(a, z) da dz$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a, z)^I = \frac{\partial c_i(a, z, j) / c_i(a, z, j)}{\partial d_{ij} / d_{ij}}, \quad \theta_{ij}(a, z)^E = \frac{\partial \pi_{ij}(a, z) / \pi_{ij}(a, z)}{\partial d_{ij} / d_{ij}},$$

and $\omega_{ij}(a, z)$ are the expenditure weights.

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and $\omega_{ij}(a, z)$ are the expenditure weights.

$$\theta_{ij}(a, z)^I = \left[- \frac{\partial a'_i(a, z, j) / p_{ij} c_i(a, z, j)}{\partial p_{ij} / p_{ij}} - 1 \right] \frac{\partial p_{ij} / p_{ij}}{\partial d_{ij} / d_{ij}}.$$

The idea: a lower d_{ij} relaxes the bc and then the division of new resources between assets and expenditure determines the intensive margin. **This is larger for the poor, smaller for the rich.**

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and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E = \frac{1}{\sigma_\epsilon} \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}.$$

Now assume the number of countries is large...

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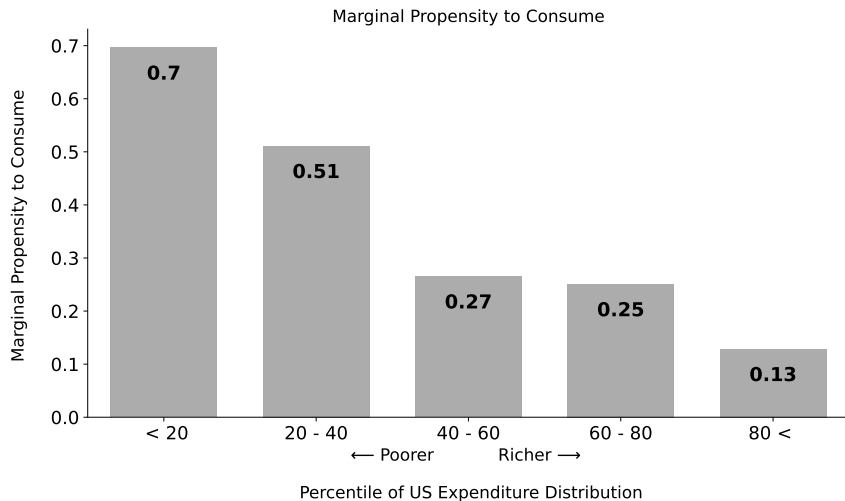
$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \quad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

$$\theta_{ij}(a,z)^E \approx -\frac{1}{\sigma_\epsilon} \left[u'(c_i(a,z,j)) c_i(a,z,j) \right].$$

With CRRA and relative risk aversion > 1 then **poor hh's are the most price sensitive on the extensive margin.**

Micro Moments — Model Consistent with HH-Level MPCs



- Household MPCs consistent with [Kaplan and Violante \(2022\)](#).

U.S. Welfare: Global 10% Reduction in d



Measuring Welfare

Want is a measure of welfare in interpretable units. I'm going to focus on equivalent variation.

Reminder: Given some price change delivering utility level v' , equivalent variation asks "at the old prices, p , how much extra income must be provided to be indifferent between v' and $v(p)$?"

My measure is a permanent, proportional increase in wealth $\tau_{i,a,z}$, at the old prices such that the new level of utility v'_i is achieved:

$$v'_i(a, z; \mathbf{p}') = v_i(a, z; \mathbf{p}, \tau_{i,a,z}).$$

Also, I'm doing this across steady states, not transitions.

Sorry :(

Preferences, Shocks, and Constraints — Calibrated Parameters

| Description | Value | Target |
|---|-------|--|
| Discount Factor, β | 0.92 | Global Interest Rate of 1% |
| CRRA parameter, γ | 1.45 | } Micro elasticities of Auer et al. (2022) |
| Type One E-V parameter, $1/\sigma_\epsilon$ | 3.0 | |
| Slope of Quality Shifter, $\psi_{ii}(z)$ | 0.72 | Micro moments of Borusyak and Jaravel (2021) |
| Borrowing Constraint ϕ_i | — | 50% of i 's autarky labor income |
| Income Process on z | — | Krueger, Mitman, and Perri (2016) |

- Everything is done under financial globalization case.

County Specific Parameters — Using Gravity as a Guide

The problem: no closed form map from trade flows to parameters as in standard trade models. But I want the model to replicate the geographic pattern of activity seen in the data.

- Step 0. Impose a trade cost function to reduce the parameter space

$$\log d_{ij} = d_k + b + l + e_h + m_i.$$

- Step 1. Run this gravity regression on the data

$$\log \left(\frac{M_{ij}}{M_{ii}} \right) = l m_i + E x_j + d_k + b + l + e_h + \delta_{ij}.$$

- Step 2. Guess TFP terms and coefficients on the trade cost function, compute an equilibrium, run the same regression from above on model generated data.
- Step 3. Evaluate difference between model and data and update parameters until convergence.

County Specific Parameters — Using Gravity as a Guide

The solution: use the gravity regression “as a guide” where I estimate parameters of the model so that the regression coefficients run on my model's data match that seen in the data.

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$$\log d_{ij} = d_k + b + l + e_h + m_i.$$

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$$\log \left(\frac{M_{ij}}{M_{ii}} \right) = l m_i + E x_j + d_k + b + l + e_h + \delta_{ij}.$$

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Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

1. Aggregate, bilateral imports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_i(a, z, j) \pi_{ij}(a, z) \lambda_i(a, z)$$

where λ_i is the *endogenous* distribution of hhs across states. Here trade flows take on a mixed-logit form similar to [Berry, Levinsohn, and Pakes \(1995\)](#), but everything is tied down in equilibrium.

2. The national income accounting identity (GDP = C + I + G + X - M) ...

$$p_i Y_i = L_i \underbrace{\sum_j \int_z \int_a p_{ij} c_i(a, z, j) \pi_{ij}(a, z) \lambda_i(a, z)}_{\bar{P}_i \bar{C}_i} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A'_i}.$$

Proposition 5 (Trade Elasticities and Welfare Gains in the Efficient Allocation)

The elasticity of trade to a change in trade costs between ij in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon} \left[u'(c_i(j)) c_i(j) \right].$$

And the welfare gains from a reduction in trade costs between i, j are

$$= \sigma_\epsilon \times \theta_{ij} \times \pi_{ij} \times \frac{L_i}{1 - \beta},$$

which is the discounted, direct effect from relaxing the aggregate resource constraint. And this can be expressed as

$$= -\sigma_\epsilon \times \frac{d\pi_{ij}/\pi_{ij}}{dd_{ij}/d_{ij}} \times \frac{L_i}{1 - \beta}.$$

Same idea as in decentralized allocation, but now everyone substitutes in a common way...

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Mimics the results of [Atkeson and Burstein \(2010\)](#) but with household (not firm) heterogeneity.

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And — again— we are back to an [Arkolakis et al. \(2012\)](#)-like expression and with log its exact.

Household Parameters

Parameters common across countries:

- CRRA for u with relative risk aversion γ — varied to fit elasticities in Auer et al. (2022).
- Earnings process as in Krueger, Mitman, and Perri (2016).
- Discount factor β juggled to target a world interest rate of 1.0% in financial globalization case.

Parameters scaled across countries to deliver balanced-growth-like properties.

- Set $\sigma_{\epsilon,i} = \sigma_{\epsilon} \times A_i^{1-\gamma}$, — σ_{ϵ} varied to fit elasticities in Auer et al. (2022).
- Set the borrowing constraint so $\phi_i = \phi \times A_i$ where $\phi = 0.50$.

Household-specific quality shifters — a home bias term $\psi_{ii}(z)$ which additively shifts utility

- Without this prices and price elasticities determine shares, so to fit the data interactions between quality and household characteristics is a way; same idea as in Berry et al. (1995).
- Slope of $\psi_{ii}(z)$ wrt z varied to fit Borusyak and Jaravel (2021) facts.

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Table 2: Estimation Results

| Barrier | Moment | HAT-Model | |
|--------------------|--------|-----------|-----------|
| | | Model Fit | Parameter |
| [0, 375) | -3.10 | -3.10 | 1.92 |
| [375, 750) | -3.67 | -3.67 | 2.39 |
| [750, 1500) | -4.03 | -4.03 | 2.64 |
| [1500, 3000) | -4.22 | -4.22 | 2.74 |
| [3000, 6000) | -6.06 | -6.06 | 4.10 |
| [6000, maximum] | -6.56 | -6.56 | 4.83 |
| Shared border | 0.30 | 0.30 | 0.92 |
| Language | 0.51 | 0.51 | 0.85 |
| EFTA | 0.04 | 0.04 | 0.96 |
| European Community | 0.54 | 0.54 | 0.91 |

Note: The first column reports data moments the HAT-model targets. The second reports the model moments. The third column reports the estimated parameter values.