Heterogeneous Agent Trade

Michael E. Waugh
Federal Reserve Bank of Minneapolis and NBER
@tradewartracker

November 25, 2024

The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. This project was developed with research support from the National Science Foundation (NSF Award number 1948800). Thomas Hasenzagl and Teerat Wongrattanapiboon provided excellent research assistance.

Heterogenous Price Elasticities and Trade

To trade economists, household heterogeneity is interesting because of the notion that some benefit from trade and others don't.

One mechanism behind this notion is heterogeneity in **elasticities**.

- Auer, Burstein, Lein, and Vogel (2022) is a nice example. In the context of the 2015 Swiss appreciation, they find that poor households are more price elastic.
- A very intuitive idea. Missing almost entirely from macro and trade, but a foundation of modern demand estimation in IO, e.g., Berry, Levinsohn, and Pakes (1995).

This paper:

- A model of household heterogeneity that results in heterogenous price elasticities and I use it as a laboratory to think about aggregate trade, the gains from trade and how they are distributed.

Heterogenous Price Elasticities and Trade — How it Works

Two ingredients:

- Trade as in Armington, but households have random utility over varieties — McFadden (1974).
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks — Bewley (1979).

Two core takeaways:

1. A model of non-homotheticities on the extensive margin with endogenous price sensitivity.

- Micro — a household's price elasticity, in essence, is about the marginal utility of wealth; under certain conditions the poor are the most price sensitive.
- Macro — bilateral trade elasticities that are non-constant, depend on the composition of demand.
- Quantitative — I calibrate the model at scale matching bilateral trade and micro facts about expenditure patterns and price elasticities ⇒ "Marshall's second law of demand" in aggregate.

Heterogenous Price Elasticities and Trade — How it Works

Two ingredients:

- Trade as in Armington, but households have random utility over varieties — McFadden (1974).
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks — Bewley (1979).

Two core takeaways:

2. The from gains from trade deviate from standard benchmarks.

- The reason is missing markets with respect to the discrete choice. Under efficiency and/or log preferences, standard results prevail.
- Quantitatively, these forces are powerful — the poorest households gain 4.5X more than the richest; the average gains from trade are 3X than representative agent benchmarks.

Related Work

HAT is part of a body of work with Simon Mongey studying general equilibrium, discrete choice economies.

- Mongey and Waugh (2024a) characterize complete markets allocations in general equilibrium, discrete choice economies.
- Mongey and Waugh (2024b) extensive margin of demand + heterogenous households + oligopoly to study markup determination and impact of fiscal transfers.

So, you might hear me say a lot of "with Simon, we..."

Two more related works:

- Fajgelbaum, Grossman, and Helpman (2011) — I can talk about differences as they arise.
- Donald, Fuku, and Miyauchi (2024) — the normative issues I raise are very similar.

Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i, competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i.

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Continuum of households $k \in [0,\ L_i]$ in each country i. Household preferences:

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \ \tilde{u}_{ijt}^k,$$

where conditional direct utility for good j is

$$\tilde{u}_{ijt}^k = u(c_{ijt}^k) + \epsilon_{jt}^k, \quad j = 1, \ldots, M.$$

Assumptions:

- discrete-continuous choice. . . so first chose one variety, then continuous choice over quantity.
- ϵ_{jt}^ks are iid across hh and time; distributed Type 1 Extreme Value with dispersion parameter σ_ϵ.
- For now, u is well behaved. For most of the characterization, I'll summarize things in terms of the hh's multipliers on its budget constraint(s).

Questions...

What about units / balanced growth?

- This is in the dispersion parameter σ_ϵ and scaling it appropriately takes care of this.

What about multiple goods at the same time?

1. Very easy to accommodate an outside good.
2. In Mongey and Waugh (2024b) we have variations with (i) continuous time or (ii) two stage budgeting with many goods, discrete varieties of a good.

What about the additivity of the shock?

- Tractable, especially with dynamics. But **does not** hardwire in non-homotheticity. Key issue is u.

In general, why?

- Settings like this are appealed to as micro-foundations of aggregators (e.g. CES).
- Evidence emphasizing extensive margin of demand as determinant of firm sales; discreetness as a key feature of household behavior in scanner data.

Model: Households II

Household k's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it}.

Households borrow or accumulate a non-state contingent asset, a, with gross return R_i. Household's face the debt limit

$$a_{t+1}^k \geq -\phi_i.$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij,t} c_{ijt}^k + a_{t+1}^k \leq R_{it} a_t^k + w_{it} z_t^k.$$

Very important object: $\lambda_{it}^k(a, z, j)$ which is the multiplier on the households j-specific budget constraint.

What Households Do I

Focus on a stationary setting. A hh's state are its asset holdings a and shock z.

1. Condition on variety choice their problem is:

$$v_i(a, z, j) = \max_{a',\ c_{ij}} \Big\{ u(c_{ij}) + \beta\, \mathbb{E}[v_i(a', z')] \Big\},$$

$$\text{subject to} \quad p_{ij} c_{ij} + a' \leq R_i a + w_i z \quad \text{and} \quad a' \geq -\phi_i.$$

2. The ex-post value function of a household in country i is

$$\max_j \{ \ v_i(a, z, j) + \epsilon_j \ \}.$$

What Households Do II

Equations characterizing the commodity choice, value functions, consumption / savings. . .

1. The choice probability is:

$$\pi_{ij}(a,z) = \exp\left(\frac{v_i(a,z,j)}{\sigma_\epsilon}\right) \Bigg/ \Phi_i(a,z), \quad \text{where} \;\; \Phi_i(a,z) := \sum_{j'} \exp\left(\frac{v_i(a,z,j')}{\sigma_\epsilon}\right).$$

2. The ex-ante value function of a household in country i is

$$v_i(a,z) = \sigma_\epsilon \log\left\{\Phi_i(a,z)\right\}.$$

3. Consumption choices must respect:

$$u'(c(a,z,j)) = \lambda_i(a,z,j)p_{ij}.$$

4. Away from the constraint, asset choices must respect:

$$\lambda_i(a,z,j) = \beta R_i \mathrm{E}_{z'}\left[\sum_{j'} \pi_{ij'}(a'(j),z')\lambda_i(a'(j),z',j')\right].$$

Equilibrium

Definition 1 (The Decentralized Stationary Equilibrium)

A Decentralized Stationary Equilibrium are policy functions and commodity choice probabilities $\{\ c_i(a,z,j), a_i'(a,z,j), \pi_{ij}(a,z)\ \}_i$, probability distributions $\{\ \psi_i(a,z)\ \}_i$ and positive real numbers $\left\{w_i, p_{ij}, R_i\right\}_{i,j}$ such that

i Prices (w_i, p_{ij}) satisfy firms problem;

ii The policy functions and choice probabilities solve the household's problem;

iii The probability distribution $\psi_i(a,z)$ induced by the policy functions, choice probabilities, and primitives satisfies the law of motion and is stationary;

iv Goods market clears:

$$p_i Y_i - \sum_j X_{ji} = 0, \quad \forall i$$

v Bond market clears with either

$$\mathrm{A}_\mathrm{i}' = 0, \quad \forall i \quad \text{or} \quad \sum_i \mathrm{A}_\mathrm{i}' = 0$$

The HA Trade Elasticity

How do i's imports from j change relative to domestic consumption due to a permanent change in d_{ij}?

Proposition 1 (The HA Trade Elasticity)

The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_{z,a} \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) da\ dz - \int_{z,a} \left\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \right\} \omega_{ii}(a,z) da\ dz$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

Super simple and intuitive — aggregates are just weighted averages of stuff at the micro level.

Next couple of slides: how the micro-elasticities and weights operate in my environment. . .

The HA Trade Elasticity

How do i's imports from j change relative to domestic consumption due to a permanent change in d_{ij}?

Proposition 1 (The HA Trade Elasticity)

The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_{z,a} \left\{ \theta_{ij}(a,z)^I + \theta_{ij}(a,z)^E \right\} \omega_{ij}(a,z) da\ dz - \int_{z,a} \left\{ \theta_{ii,j}(a,z)^I + \theta_{ii,j}(a,z)^E \right\} \omega_{ii}(a,z) da\ dz$$

which is an expenditure-weighted average of micro-level elasticities. The micro-level elasticities are decomposed into an intensive margin and extensive margin

$$\theta_{ij}(a,z)^I = \frac{\partial c_i(a,z,j)/c_i(a,z,j)}{\partial d_{ij}/d_{ij}}, \qquad \theta_{ij}(a,z)^E = \frac{\partial \pi_{ij}(a,z)/\pi_{ij}(a,z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a,z)$ are the expenditure weights.

Super simple and intuitive — aggregates are just weighted averages of stuff at the micro level.

Next couple of slides: how the micro-elasticities and weights operate in my environment. . .

The HA Trade Elasticity — Intensive Margin

In general, the extensive margin takes this form which embeds a bunch of forward looking stuff...

$$\theta_{ij}(a,z)^{I} = -\frac{1}{\gamma}\left[\frac{\partial \lambda_i(a,z,j)/\lambda_i(a,z,j)}{\partial d_{ij}/d_{ij}} + 1\right] \qquad \text{with } u \text{ being CRRA and } 1/\gamma = \text{IES}.$$

The idea: a lower d_{ij} relaxes the budget constraint and then the division of new resources between assets and expenditure determines the intensive margin.

I can bound this so

- For a very rich household, a price reduction does nothing to the budget constraint, thus it's $\frac{1}{\gamma}$.
- For a poor, hand-to-mouth, household then $\partial\lambda/\partial d_{ij}$ is $\gamma - 1 \Rightarrow$ it's one.

with CRRA and curvature parameter > 1, on the intensive margin, elasticities are **larger for the poor, smaller for the rich.**

Nice... but on the computer this is small. The extensive margin dominates.

The HA Trade Elasticity — Extensive Margin

In general, the extensive margin takes this form which embeds a bunch of forward looking stuff...

$$\theta_{ij}(a,z)^{E} = \frac{1}{\sigma_{\epsilon}} \frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}.$$

The HA Trade Elasticity — Extensive Margin

In general, the extensive margin takes this form which embeds a bunch of forward looking stuff...

$$\theta_{ij}(a,z)^{E} = \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}.$$

Now assume the number of countries is large. . .

$$\theta_{ij}(a,z)^{E} \approx -\frac{1}{\sigma_\epsilon}\Big[\lambda_i(a,z,j)c_i(a,z,j)p_{ij}\Big].$$

How this varies with wealth is a race between the marginal utility of wealth and expenditure.

The HA Trade Elasticity — Extensive Margin

In general, the extensive margin takes this form which embeds a bunch of forward looking stuff...

$$\theta_{ij}(a,z)^E = \frac{1}{\sigma_\epsilon}\frac{\partial v_i(a,z,j)}{\partial d_{ij}/d_{ij}} - \frac{\partial \Phi_i(a,z)/\Phi_i(a,z)}{\partial d_{ij}/d_{ij}}.$$

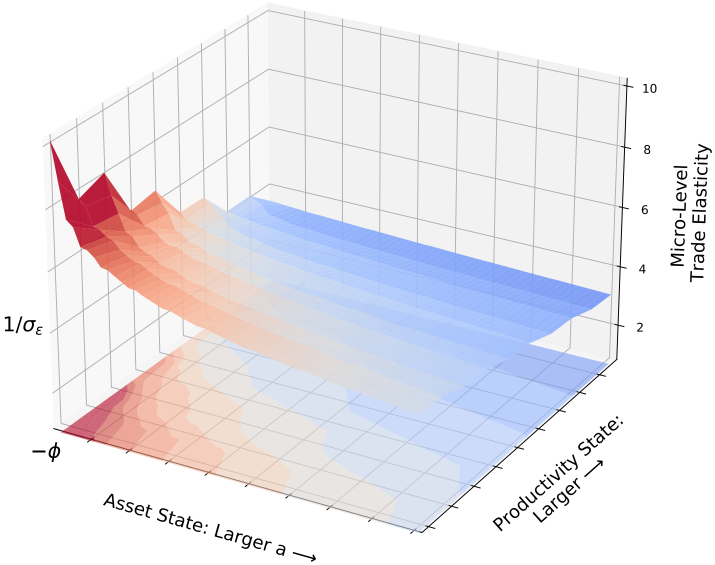
Now assume the number of countries is large **and** u is CRRA with parameter γ

$$\theta_{ij}(a,z)^E \approx -\frac{1}{\sigma_\epsilon}\left[c_i(a,z,j)^{-(\gamma-1)}\right].$$

This model easily accommodates the idea that **the poor are the most price sensitive**.

- With CRRA and curvature parameter $> 1 \Rightarrow$ elasticities are larger for the poor.
- On the computer, this is always the big, dominant component.

Two Country Example: Trade Elasticities by HH-Level State



The HA Trade Elasticity — Expenditure Weights

The aggregate, bilateral trade elasticity is a weighted average of individuals elasticities of demand.

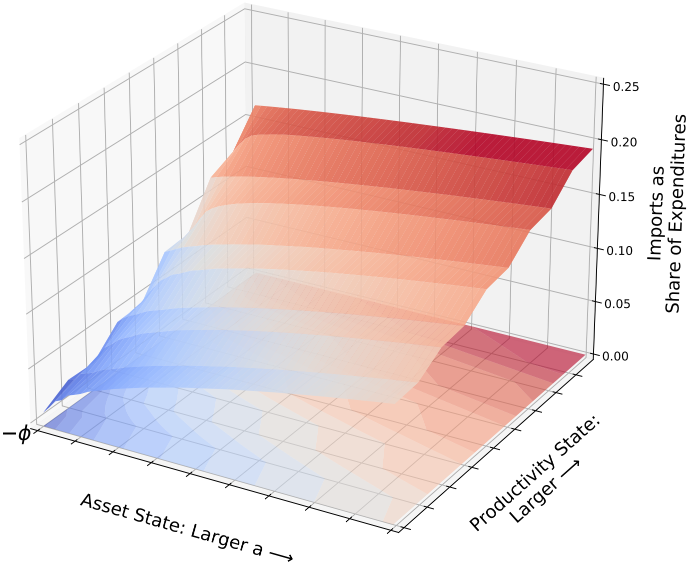
What about the weights? How these weights vary with income can be characterized by a log supermodularity condition.

- All boils down to how sensitive a household is to price — if the rich are less price sensitive **than rich hhs are more likely to purchase from high price destinations.**

Two points to remember:

1. It's a challenge relative to evidence of Borusyak and Jaravel (2021) that expenditure shares on imports are flat across income distribution.
2. It's a quantitative question as to how aggregate elasticities vary with price as expensive varieties have larger elasticities of demand for all households relative to cheaper varieties.

Two Country Example: Import Shares by HH-Level State



HA Gains from Trade

How does hh-level utility change under a partial equilibrium change in the path of price p_{ij} ?

Proposition 2 (HA Gains from Trade)

Household level gains are given by

$$\frac{\partial v_i(a,z)}{\partial p_{ij}/p_{ij}} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ -\pi_{ij}(a_t,z_t)\ \lambda(a_t,z_t,j)\ c(a_t,z_t,j)\ p_{ij} \Big\}.$$

The gains from trade = expected, discounted expenditure flow on j converted into utils at the appropriate multiplier.

The novelty is how the price sensitivity shows up (the extensive margin elasticity λcp). Conditional on exposure, price sensitivity is dictating who is gaining more or less in utils.

HA Gains from Trade

How does hh-level utility change under a partial equilibrium change in the path of price p_{ij} ?

Proposition 2 (HA Gains from Trade)

Household level gains are given by

$$\frac{\partial v_i(a,z)}{\partial p_{ij}/p_{ij}} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ -\pi_{ij}(a_t, z_t)\ \lambda(a_t, z_t, j)\ c(a_t, z_t, j)\ p_{ij} \Big\}.$$

The gains from trade = expected, discounted expenditure flow on j converted into utils at the appropriate multiplier.

The novelty is how the price sensitivity shows up (the extensive margin elasticity λcp). Conditional on exposure, price sensitivity is dictating who is gaining more or less in utils.

HA Gains from Trade — EV Units

Question: Is this really different? Yes.

To see this, put everything in date zero units of income and write it like this...

$$\frac{\partial v_i(a_0, z_0)}{\partial p_{ij}/p_{ij}} \Bigg/ \frac{\partial v_i(a_0, z_0)}{\partial y_0/y_0} = -\underbrace{\frac{c(a_0, z_0, j)\ p_{ij}}{y_0}}_{\text{"Deaton"}} \underbrace{\left[\frac{\pi_{ij}(a_0, z_0)\lambda(a_0, z_0, j)}{\sum_{j'} \pi_{ij}(a_0, z_0)\lambda(a_0, z_0, j')}\right]}_{\text{"Market Incompleteness"}} + \text{ future stuff}$$

Focusing on the date zero part, the gains in EV units are larger if

1. Expenditure share is larger... Deaton (1989), Borusyak and Jaravel (2021), many others.
2. When j is chosen, it's a state of nature where resources are scarce, high relative $\lambda(j)$.

Issue 2. vanishes if $\lambda(j)$ is equated across j. When might that be?

- Complete markets w.r.t the taste shock (Mongey and Waugh (2024a)) or efficient allocation.
- Environments where the hh can "smooth" things out, e.g., in continuous time it goes away.

HA Gains from Trade — EV Units

Question: Is this really different? Yes.

To see this, put everything in date zero units of income and write it like this...

$$\frac{\partial v_i(a_0, z_0)}{\partial p_{ij}/p_{ij}} \Bigg/ \frac{\partial v_i(a_0, z_0)}{\partial y_0/y_0} = -\underbrace{\frac{c(a_0, z_0, j)\ p_{ij}}{y_0}}_{\text{“Deaton”}} \underbrace{\left[\frac{\pi_{ij}(a_0, z_0)\lambda(a_0, z_0, j)}{\sum_{j'} \pi_{ij}(a_0, z_0)\lambda(a_0, z_0, j')}\right]}_{\text{“Market Incompleteness”}} + \text{ future stuff}$$

Focusing on the date zero part, the gains in EV units are larger if

1. Expenditure share is larger... Deaton (1989), Borusyak and Jaravel (2021), many others.
2. When j is chosen, it's a state of nature where resources are scarce, high relative $\lambda(j)$.

Issue **2.** vanishes if $\lambda(j)$ is equated across j. When might that be?

- Complete markets w.r.t the taste shock (Mongey and Waugh (2024a)) or efficient allocation.
- Environments where the hh can "smooth" things out, e.g., in continuous time it goes away.

Proposition 3 (Separation of Trade and Micro-Heterogeneity)

In the heterogenous agent trade model where preferences are logarithmic over the physical commodity, the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and trade flows satisfy a standard gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and both are independent of the household heterogeneity. And the welfare gains from trade for an individual household are

$$\frac{\partial v_i(a,z)}{\partial p_{ij}/p_{ij}} = \frac{\pi_{ij}}{(1-\beta)}$$

This mimics the results of Anderson, De Palma, and Thisse (1987). This is not obvious given the environment . . . risk, market incompleteness, borrowing constraints, etc.

HA Trade: log Preferences $\Rightarrow$ Separation of Trade and Heterogeneity

Proposition 3 (Separation of Trade and Micro-Heterogeneity)

In the heterogenous agent trade model where preferences are logarithmic over the physical commodity, the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and trade flows satisfy a standard gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and both are independent of the household heterogeneity. And the welfare gains from trade for an individual household are

$$\frac{\partial v_i(a,z)}{\partial p_{ij}/p_{ij}} = \frac{\pi_{ij}}{(1-\beta)}$$

Gains from trade simplify dramatically. And note that π_{ij} is the expenditure share.

HA Trade: log Preferences ⇒ Separation of Trade and Heterogeneity

Proposition 3 (Separation of Trade and Micro-Heterogeneity)

In the heterogenous agent trade model where preferences are logarithmic over the physical commodity, the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon},$$

and trade flows satisfy a standard gravity relationship

$$\frac{M_{ij}}{M_{ii}} = \left(\frac{w_j/A_j}{w_i/A_i}\right)^{\frac{-1}{\sigma_\epsilon}} d_{ij}^{\frac{-1}{\sigma_\epsilon}},$$

and both are independent of the household heterogeneity. And the welfare gains from trade for an individual household are

$$\frac{\partial v_i(a,z)}{\partial p_{ij}/p_{ij}} = \frac{1}{\theta(1-\beta)} \times \frac{\partial \pi_{ii}/\pi_{ii}}{\partial d_{ij}/d_{ij}}$$

And we are back to Arkolakis et al. (2012).

HA Gains from Trade under Efficiency

Proposition 4 (Trade Elasticities and Welfare Gains in the Efficient Allocation)

The elasticity of trade to a change in trade costs between ij in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\left[u'(c_i(j))c_i(j)\right].$$

And the social welfare gain from a reduction in trade costs between i, j are

$$= \sigma_\epsilon \times \theta_{ij} \times \pi_{ij} \times \frac{L_i}{1-\beta},$$

which is the discounted, direct effect from relaxing the aggregate resource constraint. And this can be expressed as

$$= -\sigma_\epsilon \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \times \frac{L_i}{1-\beta}.$$

Same idea as in decentralized allocation, but now everyone substitutes in a common way...

HA Gains from Trade under Efficiency

Proposition 4 (Trade Elasticities and Welfare Gains in the Efficient Allocation)

The elasticity of trade to a change in trade costs between ij in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\Big[u'(c_i(j))c_i(j)\Big].$$

And the social welfare gain from a reduction in trade costs between i, j are

$$= \sigma_\epsilon \times \theta_{ij} \times \pi_{ij} \times \frac{L_i}{1-\beta},$$

which is the discounted, direct effect from relaxing the aggregate resource constraint. And this can be expressed as

$$= -\sigma_\epsilon \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \times \frac{L_i}{1-\beta}.$$

Mimics the results of Atkeson and Burstein (2010) but with household (not firm) heterogeneity.

Proposition 4 (Trade Elasticities and Welfare Gains in the Efficient Allocation)

The elasticity of trade to a change in trade costs between ij in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon}\left[u'(c_i(j))c_i(j)\right].$$

And the social welfare gain from a reduction in trade costs between i, j are

$$= \sigma_\epsilon \times \theta_{ij} \times \pi_{ij} \times \frac{L_i}{1-\beta},$$

which is the discounted, direct effect from relaxing the aggregate resource constraint. And this can be expressed as

$$= -\sigma_\epsilon \times \frac{\mathrm{d}\pi_{ii}/\pi_{ii}}{\mathrm{d}d_{ij}/d_{ij}} \times \frac{L_i}{1-\beta}.$$

And — again— we are back to an Arkolakis et al. (2012)-like expression and with log its exact.

Quantitative Analysis

This is what I'll do. . .

1. Calibrate my model using my "gravity as a guide" approach on the 19 country data set of Eaton and Kortum (2002) and targeting micro-evidence from Borusyak and Jaravel (2021) and Auer et al. (2022).

2. Gains from trade calculations.

Household Parameters

Parameters common across countries:

- CRRA for u with relative risk aversion γ — varied to fit elasticities in Auer et al. (2022).
- Earnings process as in Krueger, Mitman, and Perri (2016).
- Discount factor β to target a world interest rate of 1.0% in financial globalization case.

Parameters scaled across countries to deliver balanced-growth-like properties.

- Set $\sigma_{\epsilon,i} = \sigma_\epsilon \times A_i^{1-\gamma}$, — σ_ϵ varied to fit elasticities in Auer et al. (2022).
- Set the borrowing constraint so $\phi_i = \phi \times A_i$ where $\phi = 0.50$.

Household-specific quality shifters — a home bias term $\psi_{ii}(z)$ which additively shifts utility

- Recall discussion about sorting and expenditure weights. This is a way to fit Borusyak and Jaravel (2021) facts. Currently toying with other ideas...

Preferences, Shocks, and Constraints — Calibrated Parameters

Description	Value	Target
Discount Factor, β	0.92	Global Interest Rate of 1%
CRRA parameter, γ	1.45	} Micro elasticities of Auer et al. (2022)
Type One E-V parameter, $1/\sigma_\epsilon$	3.0	
Slope of Quality Shifter, $\psi_{ii}(z)$	—	Micro moments of Borusyak and Jaravel (2021)
Borrowing Constraint ϕ_i	—	50% of i's autarky labor income
Income Process on z	—	Krueger, Mitman, and Perri (2016)

- Everything is done under financial globalization case.

County Specific Parameters — Using Gravity as a Guide

The problem: no closed form map from trade flows to parameters as in standard trade models. But I want the model to replicate the geographic pattern of activity seen in the data.

- Step 0. Impose a trade cost function to reduce the parameter space

$$\log d_{ij} = d_k + b + l + e_h + m_i.$$

- Step 1. Run this gravity regression on the data

$$\log\left(\frac{M_{ij}}{M_{ii}}\right) = Im_i + Ex_j + d_k + b + l + e_h + \delta_{ij}.$$

- Step 2. Guess TFP terms and coefficients on the trade cost function, compute an equilibrium, run the same regression from above on model generated data.
- Step 3. Evaluate difference between model and data and update parameters until convergence.

County Specific Parameters — Using Gravity as a Guide

The solution: use the gravity regression "as a guide" where I estimate parameters of the model so that the regression coefficients run on my model's data match that seen in the data.

- Step 0. Impose a trade cost function to reduce the parameter space

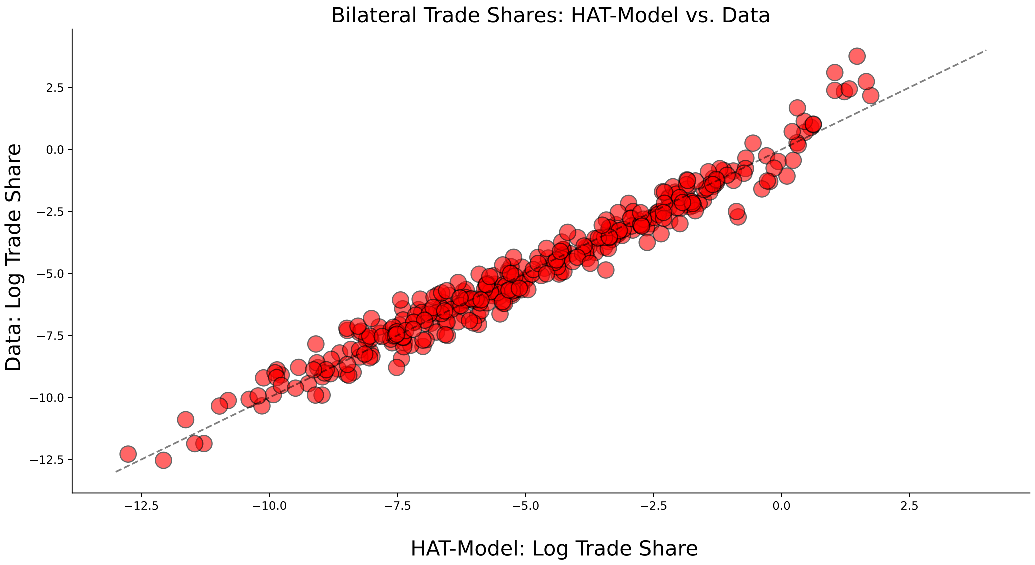
$$\log d_{ij} = d_k + b + l + e_h + m_i.$$

- Step 1. Run this gravity regression on the data

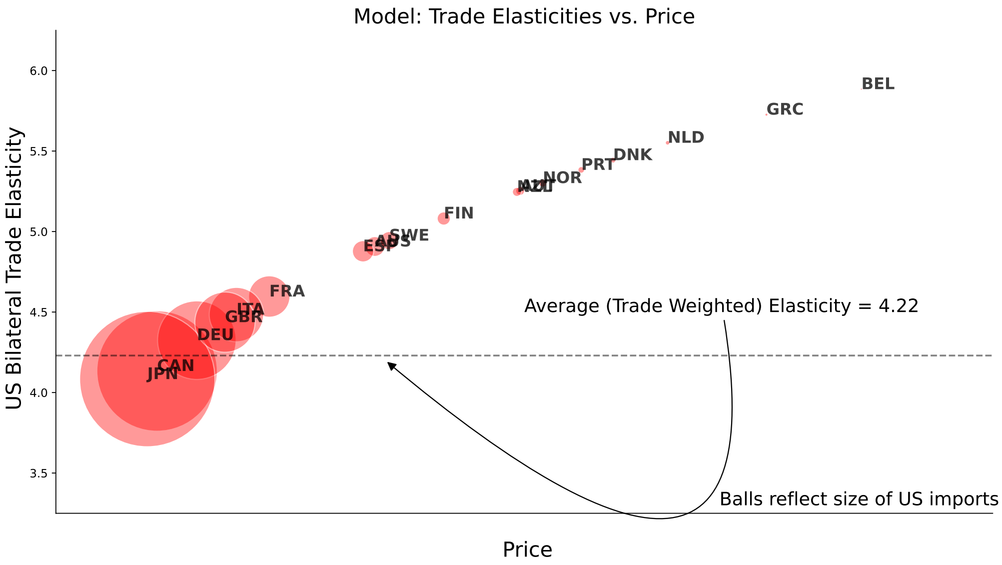
$$\log\left(\frac{M_{ij}}{M_{ii}}\right) = Im_i + Ex_j + d_k + b + l + e_h + \delta_{ij}.$$

- Step 2. Guess TFP terms and coefficients on the trade cost function, compute an equilibrium, run the same regression from above on model generated data.
- Step 3. Evaluate difference between model and data and update parameters until convergence.

Bilateral Trade: Model vs. Data



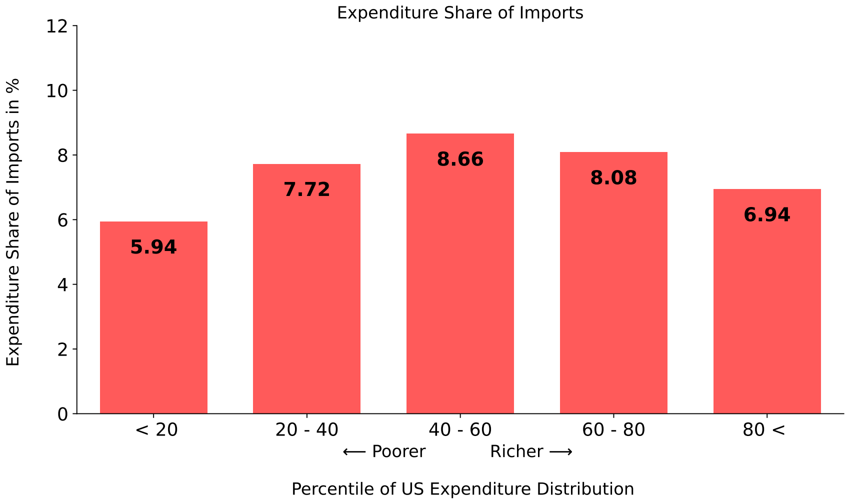
US Trade Elasticities: $-\theta_{us,j}$



Micro Moments — Model Consistent with HH-Level Elasticities

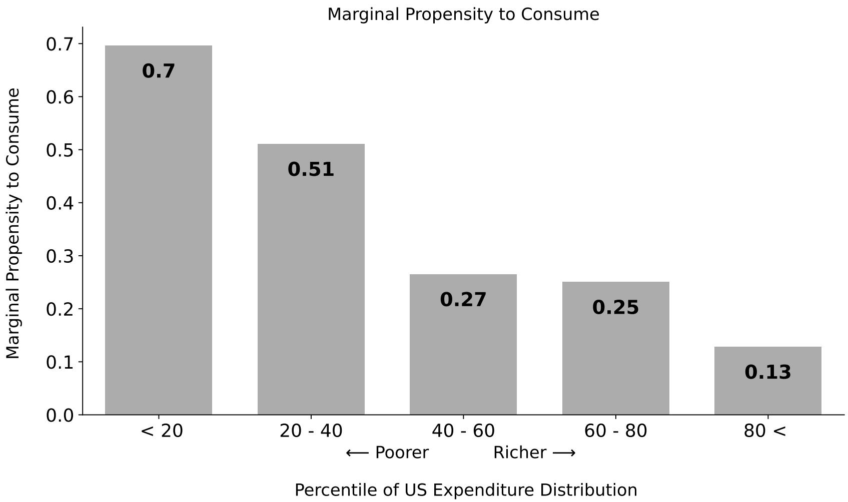
- Household-level elasticities consistent with those in Auer, Burstein, Lein, and Vogel (2022), i.e. rich less elastic than the poor.

Micro Moments — Model Consistent with HH-Level Expenditure Patterns



- Household-level import shares consistent with facts from Borusyak and Jaravel (2021), i.e. rich and poor do not spend unequally on imports.

Micro Moments — Model Consistent with HH-Level MPCs



- Household MPCs consistent with Kaplan and Violante (2022).

Measuring Welfare

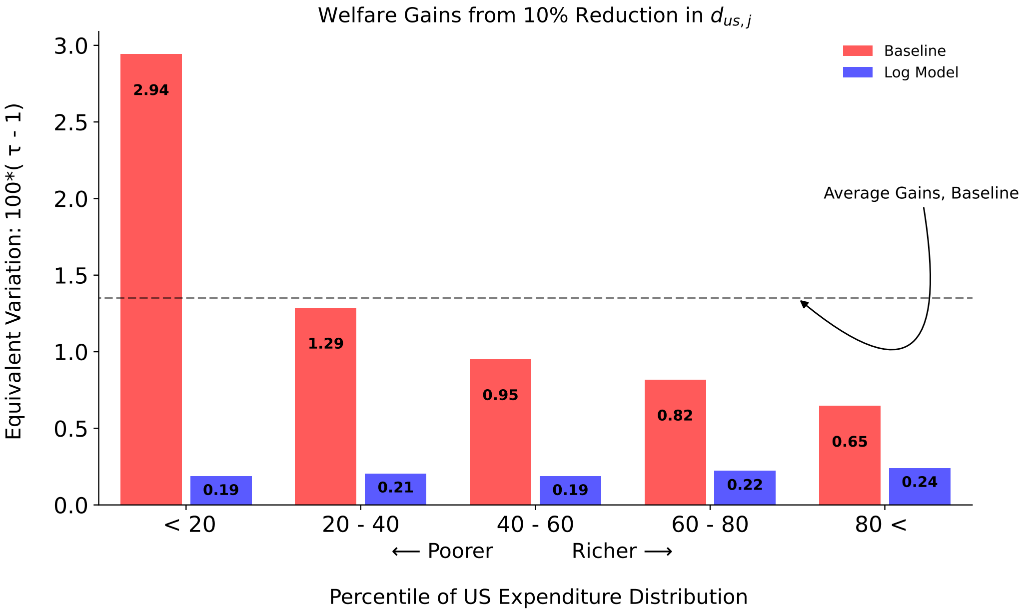
Today: My measure is a permanent, proportional increase in wealth $\tau_{i,a,z}$, at the old prices such that the new level of utility v_i' is achieved:

$$v_i'(a, z; \mathbf{p}') = v_i(a, z; \mathbf{p}, \tau_{i,a,z}).$$

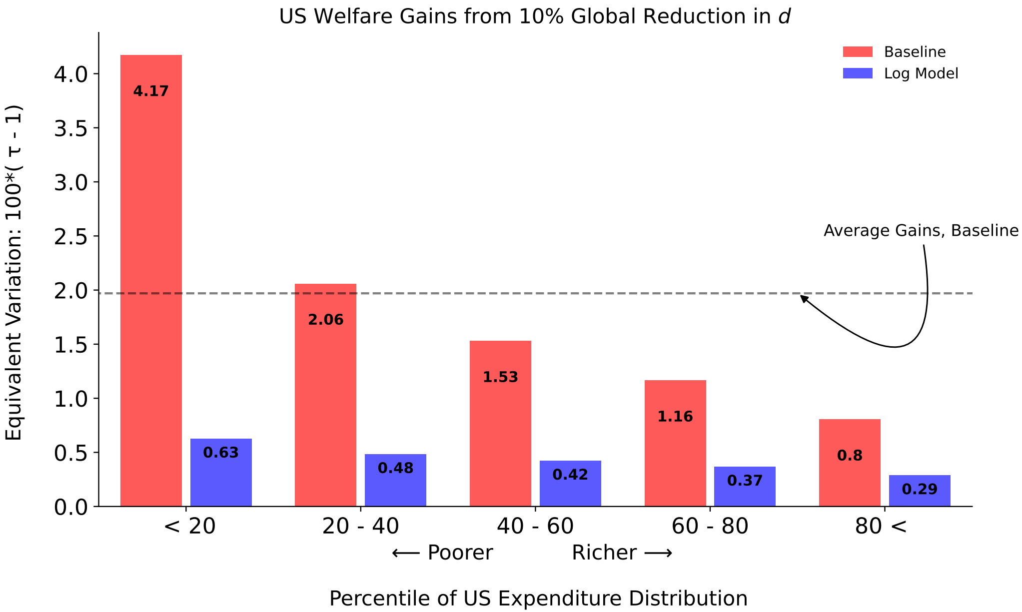
I'm doing this across steady states, not transition path.

- I've computed the path recently, need more time to think about results... sorry :(

U.S. Welfare: 10% Reduction in $d_{us,j}$



U.S. Welfare: Global 10% Reduction in d



Final Thoughts...

This paper has prompted even more questions...

- The efficient pattern of trade? In a companion paper, I show that "near-shoring" is an outcome that a global planner likes.
- Can trade policy improve outcomes? Put in tariffs and redistribute?
- The interaction between trade goods and trade in assets?

One more thing: My github repository provides the code and supplementary work behind this paper at `https://github.com/mwaugh0328/heterogeneous-agent-trade`.

References I

Anderson, S. P., A. De Palma, and J.-F. Thisse (1987): "The CES is a discrete choice model?,," *Economics Letters*, 24, 139–140.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.

Atkeson, A. and A. T. Burstein (2010): "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, 118, 433–484.

Auer, R., A. Burstein, S. M. Lein, and J. Vogel (2022): "Unequal expenditure switching: Evidence from Switzerland," Tech. rep., National Bureau of Economic Research.

Berry, S., J. Levinsohn, and A. Pakes (1995): "Automobile prices in market equilibrium," *Econometrica*, 63, 841–890.

Bewley, T. (1979): "The optimum quantity of money," *Discussion Paper*.

Borusyak, K. and X. Jaravel (2021): "The distributional effects of trade: Theory and evidence from the united states," Tech. rep., National Bureau of Economic Research.

Deaton, A. (1989): "Rice prices and income distribution in Thailand: A non-parametric analysis," *Economic Journal*, 99, 1–37.

Donald, E., M. Fuku, and Y. Miyauchi (2024): "Unpacking Aggregate Welfare in a Spatial Economy," Tech. rep.

Eaton, J. and S. Kortum (2002): "Technology, geography, and trade," *Econometrica*, 70, 1741–1779.

Fajgelbaum, P., G. M. Grossman, and E. Helpman (2011): "Income distribution, product quality, and international trade," *Journal of Political Economy*, 119, 721–765.

Kaplan, G. and G. L. Violante (2022): "The marginal propensity to consume in heterogeneous agent models," *Annual Review of Economics*, 14, 747–775.

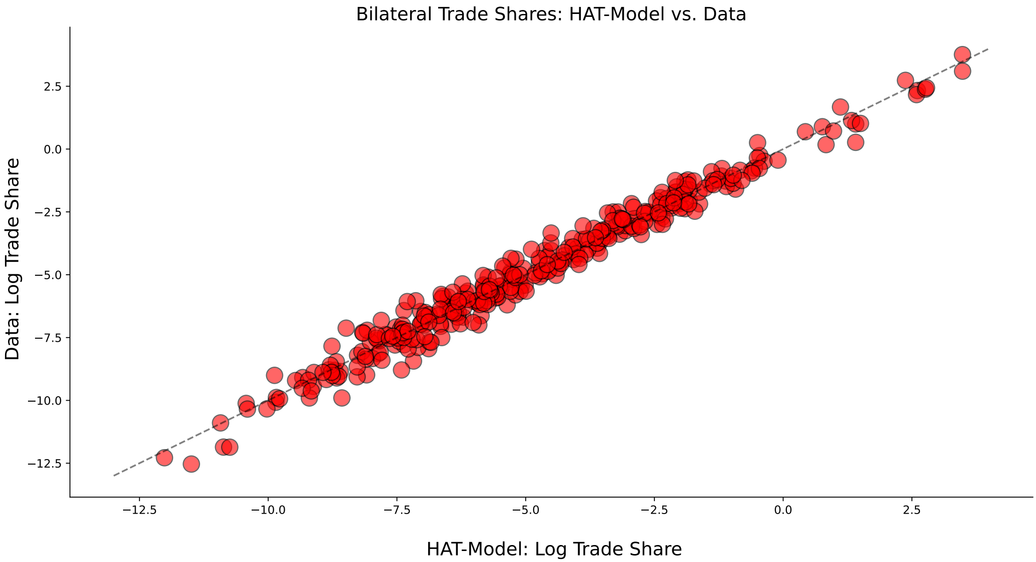
Krueger, D., K. Mitman, and F. Perri (2016): "Macroeconomics and household heterogeneity," in *Handbook of Macroeconomics*, ed. by H. U. J. B. Taylor, Elsevier, vol. 2, 843–921.

McFadden, D. (1974): "Conditional logit analysis of qualitative choice behavior," in *Frontiers in Econometrics, edited by P. Zarembka, 105-142*, Academic Press.

Mongey, S. and M. Waugh (2024a): "Discrete choice, complete markets, and equilibrium," .

——— (2024b): "Pricing Inequality," .

Log Model — Fit of Trade Data



Measuring Welfare

Want is a measure of welfare in interpretable units. I'm going to focus on equivalent variation.

Reminder: Given some price change delivering utility level v', equivalent variation asks "at the old prices, p, how much extra income must be provided to be indifferent between v' and $v(p)$?"

My measure is a permanent, proportional increase in wealth $\tau_{i,a,z}$, at the old prices such that the new level of utility v_i' is achieved:

$$v_i'(a, z; \mathbf{p}') = v_i(a, z; \mathbf{p}, \tau_{i,a,z}).$$

Also, I'm doing this across steady states, not transitions.
Sorry :(

Preferences, Shocks, and Constraints — Calibrated Parameters

Description	Value	Target
Discount Factor, β	0.92	Global Interest Rate of 1%
CRRA parameter, γ	1.45	} Micro elasticities of Auer et al. (2022)
Type One E-V parameter, $1/\sigma_\epsilon$	3.0	
Slope of Quality Shifter, $\psi_{ii}(z)$	0.72	Micro moments of Borusyak and Jaravel (2021)
Borrowing Constraint ϕ_i	—	50% of i's autarky labor income
Income Process on z	—	Krueger, Mitman, and Perri (2016)

- Everything is done under financial globalization case.

Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

1. Aggregate, bilateral imports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_i(a, z, j) \pi_{ij}(a, z) \lambda_i(a, z)$$

where λ_i is the *endogenous* distribution of hhs across states. Here trade flows take on a mixed-logit form similar to Berry, Levinsohn, and Pakes (1995), but everything is tied down in equilibrium.

2. The national income accounting identity (GDP = C + I + G + X - M) . . .

$$p_i Y_i = \underbrace{L_i \sum_j \int_z \int_a p_{ij} c_i(a, z, j) \pi_{ij}(a, z) \lambda_i(a, z)}_{\widetilde{P_i C_i}} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A_i'}.$$

Table 3: Estimation Results

Barrier	Moment	HAT-Model Model Fit	HAT-Model Parameter
$[0, 375)$	-3.10	-3.10	1.92
$[375, 750)$	-3.67	-3.67	2.39
$[750, 1500)$	-4.03	-4.03	2.64
$[1500, 3000)$	-4.22	-4.22	2.74
$[3000, 6000)$	-6.06	-6.06	4.10
$[6000, \text{maximum}]$	-6.56	-6.56	4.83
Shared border	0.30	0.30	0.92
Language	0.51	0.51	0.85
EFTA	0.04	0.04	0.96
European Community	0.54	0.54	0.91

Note: The first column reports data moments the HAT-model targets. The second reports the model moments. The third column reports the estimated parameter values.