

Heterogeneous Agent Trade

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[@tradewartracker](#)

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What am I doing?

Big picture — these are the questions that interest me. . .

1. What are distributional consequences of trade?
2. Is there a role for trade policy to improve outcomes?

One mechanism behind **1.** is heterogeneity in expenditure shares on traded goods and **elasticities**.

- [Auer, Burstein, Lein, and Vogel \(2022\)](#) is a nice example. In the context of the 2015 Swiss appreciation, they find that poor households are more price elastic.

Behind **2.** are inefficiencies arising from market incompleteness that feed into product markets. . . so lack of insurance against life's circumstances distorts the pattern of trade.

This paper:

GE, heterogenous agent model of trade delivering ABLV-like facts. I work out the implications for aggregate trade, the gains from trade, and the normative implications.

How I do it...

Two ingredients:

- Trade as in Armington, but households have random utility over these varieties.
- Standard incomplete markets model with households facing incomplete insurance against idiosyncratic productivity and taste shocks.

Qualitatively I characterize...

- How price elasticities vary at the micro-level and when micro-heterogeneity shapes aggregates.
- The welfare gains from trade at the micro and macro level.
- The efficient allocation and, thus, how market incompleteness shapes these outcomes.

Quantitatively, I compute a 19 country model (the [Eaton and Kortum \(2002\)](#) data) and... today, gains from trade and the role of the asset market, and a little bit about the planner.

Model: Production and Trade

M countries. Each country produces a nationally differentiated product as in Armington.

In country i , competitive firms' produce variety i with:

$$Q_i = A_i N_i,$$

where A_i is TFP; N_i are efficiency units of labor supplied by households.

Cross-country trade faces obstacles:

- iceberg trade costs $d_{ij} > 1$ for one unit from supplier j to go to buyer i .

This structure leads to the following prices that households face

$$p_{ij} = \frac{d_{ij} w_j}{A_j}.$$

Model: Households I

Continuum of households $k \in [0, L_i]$ in each country i . Household preferences:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \tilde{u}_{ijt}^k$$

where conditional direct utility for good j is

$$\tilde{u}_{ijt}^k = u(c_{ijt}^k) + \epsilon_{jt}^k, \quad j = 1, \dots, M$$

Assumptions:

- Two-stage budgeting as in [Anderson, De Palma, and Thisse \(1987\)](#)... so first chose variety, then continuous choice over quantity.
- ϵ_{jt}^k s are iid across hh and time; distributed Type 1 Extreme Value with dispersion parameter σ_ϵ .
- For now, I only assume u is well behaved.

Alternative interpretation — the “infinite shopping aisle” we use in [Mongey and Waugh \(2022\)](#).

Model: Households II

Household k 's efficiency units z_t evolve according to a Markov Chain. They face the wage per efficiency unit w_{it} .

Households borrow or accumulate a non-state contingent asset, a , with gross return R_i . Household's face the debt limit

$$a_{t+1} \geq -\phi_i.$$

Conditional on a variety choice, a household's budget constraint is

$$p_{ij}c_{ijt} + a_{t+1} \leq R_i a_t + w_{it}z_t.$$

What Households Do I

Focus on a stationary setting. A hh's state are its asset holdings a and shock z .

1. Condition on variety choice their problem is:

$$v_{ij}(a, z) = \max_{a', c_{ij}} \left\{ u(c_{ij}) + \epsilon_j + \beta \mathbb{E}[v_i(a', z')] \right\}$$

$$\text{subject to } p_{ij}c_{ij} + a' \leq R_i a + w_i z \quad \text{and} \quad a' \geq -\phi_i.$$

2. The ex-post value function of a household in country i is

$$v_i(a, z) = \max_j \{ v_{ij}(a, z) \}.$$

What Households Do II

Two equations characterizing the commodity choice, consumption / savings. . .

1. The choice probability is:

$$\pi_{ij}(a, z) = \exp\left(\frac{v_{ij}(a, z)}{\sigma_\epsilon}\right) / \Phi_i(a, z)$$

$$\text{where } \Phi_i(a, z) := \sum_{j'} \exp\left(\frac{v_{ij'}(a, z)}{\sigma_\epsilon}\right)$$

2. Away from the constraint, consumption and asset choices must respect this Euler Equation:

$$\frac{u'(c_{ij}(a, z))}{p_{ij}} = \beta R_i \mathbb{E}_{z'} \left[\sum_{j'} \pi_{ij'}(a', z') \frac{u'(c_{ij'}(a', z'))}{p_{ij'}} \right].$$

Aggregation

Aggregates arise from explicit aggregation of hh-level actions. Two examples:

1. Aggregate, bilateral imports and exports are

$$M_{ij} = L_i \int_z \int_a p_{ij} c_{ij}(a, z) \pi_{ij}(a, z) \lambda_i(a, z), \quad X_{ji} = L_j \int_z \int_a p_{ji} c_{ji}(a, z) \pi_{ji}(a, z) \lambda_j(a, z),$$

where λ_i is the distribution of hhs across states and $c_{ij}(a, z)$ is the consumption function. Here trade flows take on a mixed logit formulation similar to [Berry, Levinsohn, and Pakes \(1995\)](#).

2. The national income accounting identity (GDP = C + I + G + X - M) ...

$$p_i Y_i = L_i \underbrace{\sum_j \int_z \int_a p_{ij} c_{ij}(a, z) \pi_{ij}(a, z) \lambda_i(a, z)}_{\widetilde{P}_i \widetilde{C}_i} + \underbrace{\left[\sum_{j \neq i} X_{ji} - \sum_{j \neq i} M_{ij} \right]}_{-R_i A_i + A'_i}.$$

Equilibrium

The Decentralized Stationary Equilibrium. A Decentralized Stationary Equilibrium are asset policy functions and commodity choice probabilities $\{g_{ij}(a, z), \pi_{ij}(a, z)\}_{ij}$, probability distributions $\{\lambda_i(a, z)\}_i$ and positive real numbers $\{w_i, p_{ij}, R_i\}_{ij}$ such that

- i Prices (w_i, p_{ij}) satisfy the firms problem;
- ii The policy functions and choice probabilities solve the household's optimization problem;
- iv The probability distribution $\lambda_i(a, z)$ induced by the policy functions, choice probabilities, and primitives satisfies the law of motion and is stationary;
- v Goods market clears:

$$p_i Y_i - \sum_j^M X_{ji} = 0, \quad \forall i$$

- v Bond market clears with either

$A'_i = 0, \quad \forall i$ in the case of financial autarky, or

$$\sum_i A'_i = 0, \text{ in the case of financial globalization and } R_i = R \quad \forall i$$

Proposition #1: The H-A Trade Elasticity. The trade elasticity between country i and country j is:

$$\theta_{ij} = 1 + \int_a \int_z \left\{ \theta_{ij}(a, z)^I + \theta_{ij}(a, z)^E \right\} \omega_{ij}(a, z) - \left\{ \theta_{ii}(a, z)^I + \theta_{ii}(a, z)^E \right\} \omega_{ii}(a, z),$$

which is the difference between ij and ii expenditure-weighted micro-level elasticities. The micro-level elasticities for households with states a, z are an intensive and extensive elasticity

$$\theta_{ij}(a, z)^I = \frac{\partial c_{ij}(a, z)/c_{ij}(a, z)}{\partial d_{ij}/d_{ij}}, \quad \theta_{ij}(a, z)^E = \frac{\partial \pi_{ij}(a, z)/\pi_{ij}(a, z)}{\partial d_{ij}/d_{ij}},$$

and $\omega_{ij}(a, z)$ are the expenditure weights.

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and $\omega_{ij}(a, z)$ are the expenditure weights.

$$\theta_{ij}(a, z)^I = \left[- \frac{\partial g_{ij}(a, z) / p_{ij} c_{ij}(a, z)}{\partial p_{ij} / p_{ij}} - 1 \right] \frac{\partial p_{ij} / p_{ij}}{\partial d_{ij} / d_{ij}}.$$

The idea here is that reduction in trade costs relaxes the hh's budget constraint and then the division of new resources between assets and expenditure determines the intensive margin elasticity.

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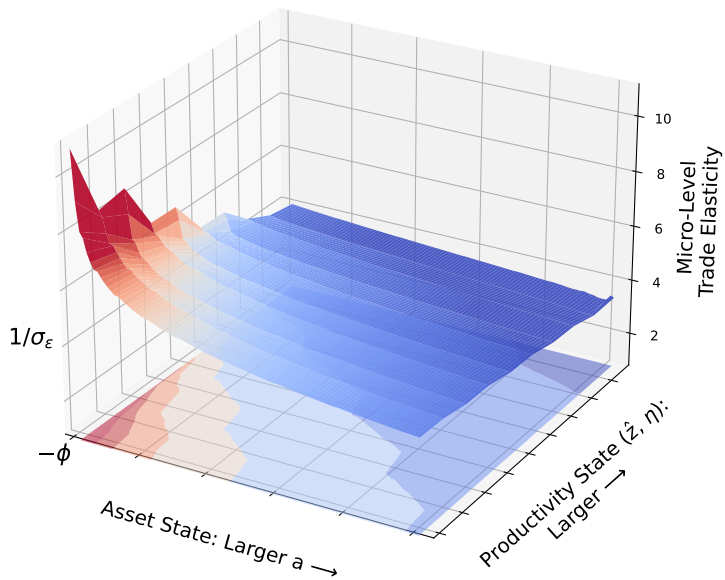
and $\omega_{ij}(a, z)$ are the expenditure weights.

$$\theta_{ij}(a, z)^E = -\frac{\partial \Phi_i(a, z)/\Phi_i(a, z)}{\partial d_{ij}/d_{ij}} + \frac{1}{\sigma_\epsilon} \frac{\partial v_{ij}(a, z)}{\partial d_{ij}/d_{ij}}.$$

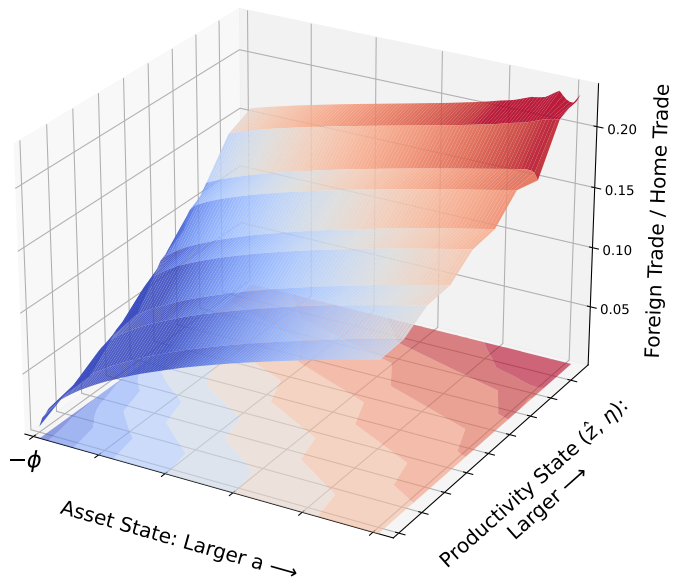
Key is $\frac{\partial v_{ij}(a, z)}{\partial d_{ij}/d_{ij}}$.

In the paper, I show that if Arrow-Pratt measure of relative risk aversion > 1 than hh's with (i) high $u'(c)$ and (ii) high MPCs are more price elastic. **So poor hh's are the most price sensitive.**

Trade Elasticities by HH-Level State



Trade Shares: $M_{ij}(a, z)/M_{ii}(a, z)$, by HH-Level State



Proposition #2: H-A Welfare Gains from Trade. The gains from trade are

$$\frac{dW_i}{dd_{ij}/d_{ij}} \approx \int_z \int_a \left\{ \underbrace{\frac{dv_i(a, z)}{dd_{ij}/d_{ij}}}_{\text{gains to hh}} + v_i(a, z) \underbrace{\frac{d\lambda_i(a, z)/\lambda_i(a, z)}{dd_{ij}/d_{ij}}}_{\text{gains to reallocation}} \right\} L_i \lambda_i(a, z),$$

where v_i is value function before realization of taste shocks; \approx is about abstracting from transition.

Household-level gains are

$$\frac{dv_i(a, z)}{dd_{ij}/d_{ij}} \approx \sigma_\epsilon \frac{d\Phi_i(a, z)/\Phi_i(a, z)}{dd_{ij}/d_{ij}}$$

Just like [Eaton and Kortum \(2002\)](#)! It's all about how this price-index-like thing changes.

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where v_i is value function before realization of taste shocks; \approx is about abstracting from transition.

Household-level gains are

$$\frac{dv_i(a, z)}{dd_{ij}/d_{ij}} \approx \sum_j \pi_{ij}(a, z) \frac{dv_{ij}(a, z)}{dd_{ij}/d_{ij}}$$

The change in the $\Phi_i(a, z)$ thing (previous slide if you fell asleep) is share-weighted average of choice-specific value functions.

Next step... one algebra trick.

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where v_i is value function before realization of taste shocks; \approx is about abstracting from transition.

Household-level gains are

$$\frac{dv_i(a, z)}{dd_{ij}/d_{ij}} \approx \underbrace{\sum_j \pi_{ij}(a, z) \left\{ \frac{dv_{ij}(a, z)}{dd_{ij}/d_{ij}} - \frac{dv_{ii}(a, z)}{dd_{ij}/d_{ij}} \right\}}_{\text{how relative valuations change}} + \frac{dv_{ii}(a, z)}{dd_{ij}/d_{ij}}$$

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Now recursively iterate forward in time given how v_{ii} connects with v_i next period.

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where v_i is value function before realization of taste shocks; \approx is about abstracting from transition.

Household-level gains are

$$\frac{dv_i(a, z)}{dd_{ij}/d_{ij}} \approx \mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \left\{ -\sigma_\epsilon \frac{d\pi_{ii}(a_t, z_t)/\pi_{ii}(a_t, z_t)}{dd_{ij}/d_{ij}} + u'(c_{ii}(a_t, z_t)) \left[a_t \frac{dR_i/p_{ii}}{dd_{ij}/d_{ij}} \right] \right\}.$$

HH-level gains pick up two effects:

- An ACR-like term summarizing how relative valuations across choices change.
- How hh's real wealth (+ or -) change through GE effects on prices — all evaluated at the hh's marginal utility of consumption.

Proposition #3: Separation of Trade and Micro-Heterogeneity. When preferences are logarithmic over the physical commodity, choice probabilities are independent of household heterogeneity

$$\pi_{ij}(a, z) = \exp\left(\frac{-\log p_{ij}}{\sigma_\epsilon}\right) \bigg/ \sum_{j'} \exp\left(\frac{-\log p_{ij'}}{\sigma_\epsilon}\right),$$

and the trade elasticity is

$$\theta = -\frac{1}{\sigma_\epsilon}.$$

And hh-level gains from trade

$$\frac{dv_i(a, z)}{dd_{ij}/d_{ij}} \approx \underbrace{-\frac{1}{\theta(1-\beta)} \times \frac{d\pi_{ii}/\pi_{ii}}{dd_{ij}/d_{ij}}}_{\text{ACR}} + \underbrace{\mathbb{E}_z \sum_{t=0}^{\infty} \beta^t \left[u'(c_{ii}(a_t, z_t)) a_t \frac{dR_i/p_{ii}}{dd_{ij}/d_{ij}} \right]}_{\text{with the magic of log, turns out this = 0}}.$$

This mimics the results of [Anderson, De Palma, and Thisse \(1987\)](#). This was not obvious to me given the environment . . . risk, market incompleteness, borrowing constraints, etc.

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And we are back to [Arkolakis et al. \(2012\)](#)...

Proposition #4: The Centralized (Efficient) Allocation. The allocation satisfying the Centralized Planning Problem (with a utilitarian SWF and country-specific Pareto weights ψ_i) is:

1. An allocation of consumption satisfying:

$$\psi_i u'(c_{ij}(z, t)) = \chi_j(t) d_{ij}$$

where $\chi_j(t)$ is the multiplier on j resource constraint for variety j ,

2. And variety choice probabilities:

$$\pi_{ij}(t) = \exp\left(\frac{u(c_{ij}(t)) - u'(c_{ij}(t))c_{ij}(t)}{\sigma_\epsilon}\right) / \sum_{j'} \exp\left(\frac{u(c_{ij'}(t)) - u'(c_{ij'}(t))c_{ij'}(t)}{\sigma_\epsilon}\right).$$

1. is a [Backus and Smith \(1993\)](#)-like condition.

2. is new — trade should reflect the net social benefit of buying that commodity.

Proposition #5: Trade Elasticities and Welfare Gains in the Efficient Allocation The trade elasticity between i, j in the efficient allocation is:

$$\theta_{ij} = -\frac{1}{\sigma_\epsilon} \left[u'(c_{ij}) c_{ij} \right].$$

And the welfare gains from a reduction in trade costs between i, j are

$$\frac{dW}{dd_{ij}/d_{ij}} = \frac{\partial W}{\partial d_{ij}/d_{ij}} = \frac{\psi_i}{1 - \beta} \times u'(c_{ij}) c_{ij} \pi_{ij} L_i,$$

which is the discounted, weighted, direct effect from relaxing the resource constraint.

Mimics the results of [Atkeson and Burstein \(2010\)](#) but with household (not firm) heterogeneity. With log preferences the direct effect is equivalent to [Arkolakis et al. \(2012\)](#).

Quantitative Analysis

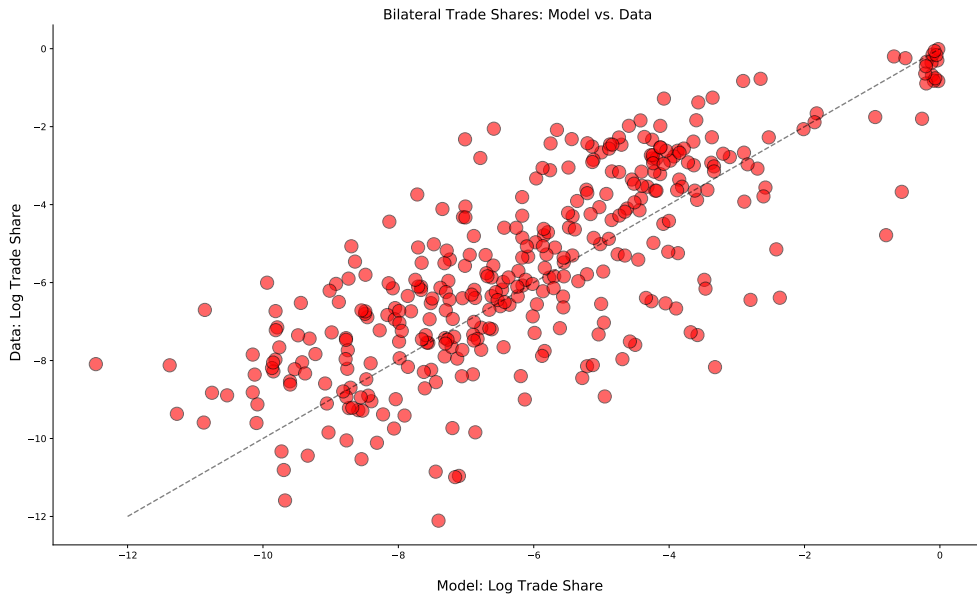
Still preliminary. This is what I'm going to do:

- Grab trade costs and productivity estimates from 19 country world of [Eaton and Kortum \(2002\)](#) and compute an equilibrium.
- Explore bilateral reduction in trade costs...I'll explain in two slides.
- Do all of this in Financial Globalization case...no balance of trade.

Other important parameters and how I set them for today.

- Taste shock parameter so $1/\sigma_\epsilon = 4.0$. CRRA for u with relative risk aversion = 1.5.
- Earnings process is a mixture of a persistent and transitory component and calibrated as in [Krueger, Mitman, and Perri \(2016\)](#).
- Borrowing constraint is set $\approx 2\times$ earnings for US. Discount factor set so $R \approx 2\%$ for US.

Bilateral Trade: Model vs. Data



Taking the Model for a Ride

Two ideas I want to illustrate:

1. You pick the market, you pick a person.

- Rich vs. poor benefit differently depending upon the market.

2. GE effects create winners and losers

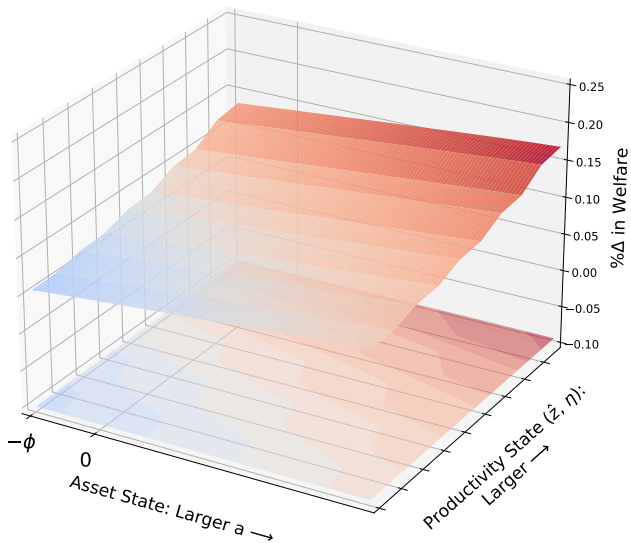
- Who benefits from **1.** + effects on $R/p \Rightarrow$ shapes the extent to which they are winners and losers.

Next slides: 10% reduction to US import trade cost on different source markets. . . Japan, Canada.

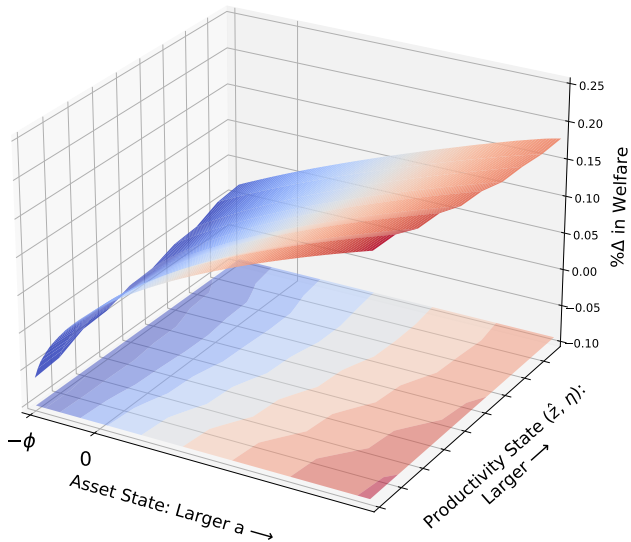
Focus on US welfare and break it down by

- A. Fix R & w , so what is direct effect of change in trade cost,
- B. R & w adjust to clear goods and asset markets.

U.S. Welfare: 10% Reduction to Japan, Fixed R & w



U.S. Welfare: 10% Reduction to Japan, GE

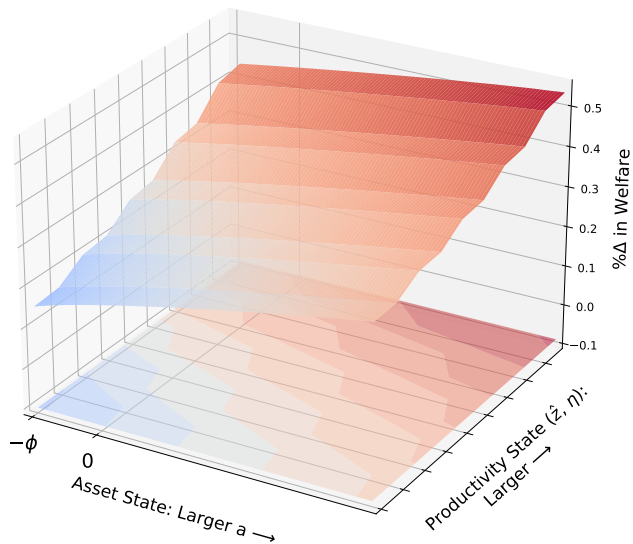


U.S. Welfare: 10% Reduction to Japan

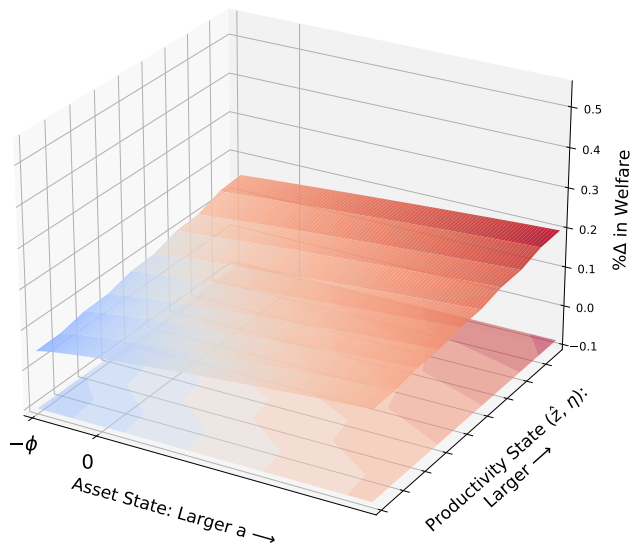
Welfare by Wealth — Japan 10% Reduction

Asset Quartile	Fixed R & w	GE: Prices Adjust
	Welfare (% Change)	Welfare (% Change)
Bottom quartile	0.06	-0.028
Median	0.08	0.02
Upper quartile	0.011	0.10
Aggregate	0.09	0.02
% losers	0.0	37.1

U.S. Welfare: 10% Reduction to Canada, Fixed R & w



U.S. Welfare: 10% Reduction to Canada, GE



U.S. Welfare: 10% Reduction to Canada

Welfare by Wealth — Canada 10% Reduction

Asset Quartile	Fixed R & w	GE: Prices Adjust
	Welfare (% Change)	Welfare (% Change)
Bottom quartile	0.21	0.07
Median	0.28	0.09
Upper quartile	0.39	0.13
Aggregate	0.30	0.10
% losers	0.0	0.0

Recap...

Two ideas:

1. You pick the market, you pick a person.

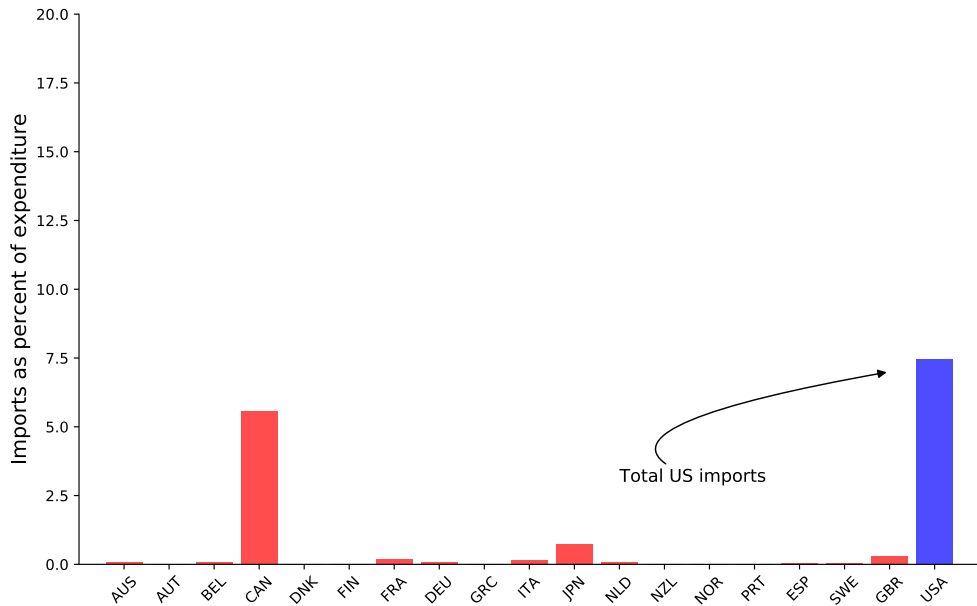
2. GE effects create winners and losers

1 + 2 leads to the idea that the being exposed / liberalizing with some markets is better than others.

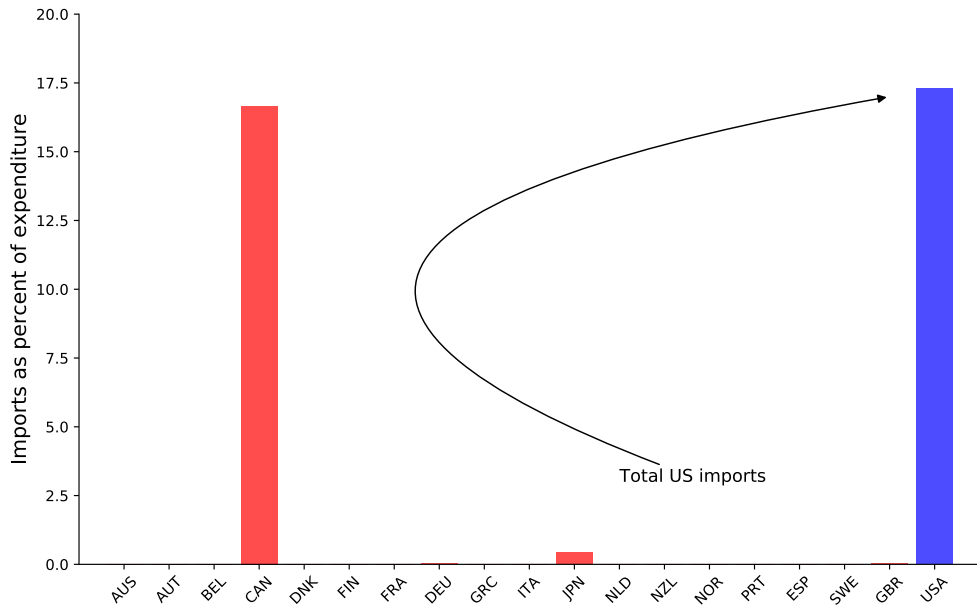
This shows up in the planners allocation. Next two slides..

- Trade in the decentralized equilibrium vs.
- the Planner's allocation

US Trade in the Decentralized Equilibrium



US Trade in the Planner's Allocation



Where I'm headed next...

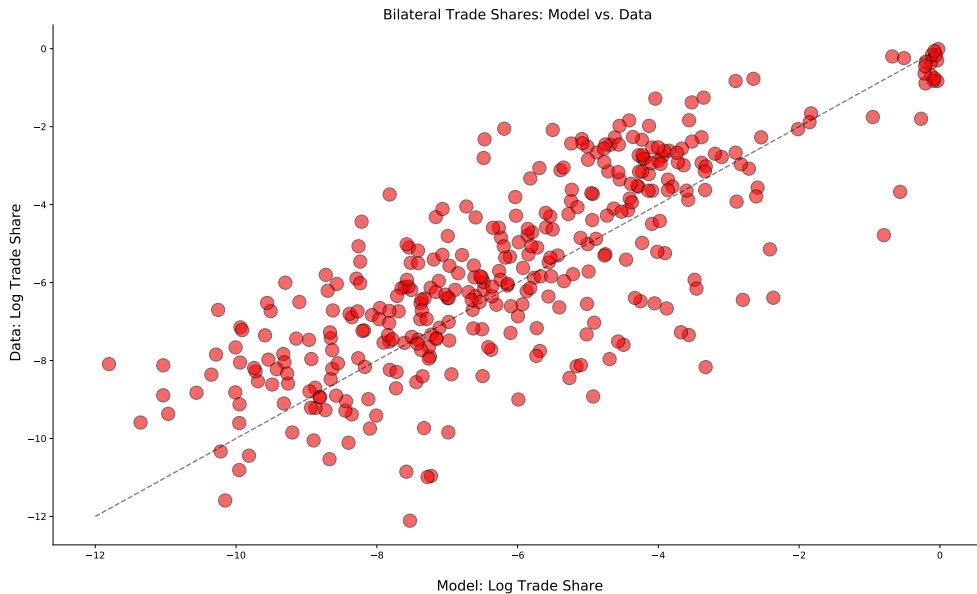
Lot's to do, but "big picture" this is where I'm aiming:

1. Use gravity model + indirect inference to estimate the model. And better confront micro-evidence.
 - "Gravity as a guide, not as a law"
2. Can trade policy improve outcomes?
 - Put tariffs in and redistribute!

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Financial Autarky, Bilateral Trade: Model vs. Data



Micro-Elasticities I: The Intensive Margin

How do households respond on the **intensive** margin to a change in trade costs?

$$\begin{aligned}\theta_{ij}(a, z)' &:= \frac{\partial c_{ij}(a, z)/c_{ij}(a, z)}{\partial d_{ij}/d_{ij}}, \\ &= \left[- \frac{\partial g_{ij}(a, z)/p_{ij}c_{ij}(a, z)}{\partial p_{ij}/p_{ij}} - 1 \right] \frac{\partial p_{ij}/p_{ij}}{\partial d_{ij}/d_{ij}}.\end{aligned}$$

The idea: A reduction in trade costs relaxes the hh's budget constraint, so the intensive margin elasticity depends on the division of new resources between assets and expenditure.

Micro-Elasticities II: The Extensive Margin

How do households respond on the **extensive** margin?

$$\begin{aligned}\theta_{ij}(\mathbf{a}, z)^E &:= \frac{\partial \pi_{ij}(\mathbf{a}, z) / \pi_{ij}(\mathbf{a}, z)}{\partial d_{ij} / d_{ij}}, \\ &= -\frac{\partial \Phi_i(\mathbf{a}, z) / \Phi_i(\mathbf{a}, z)}{\partial d_{ij} / d_{ij}} - \frac{1}{\sigma_\epsilon} \left[u'(c_{ij}(\mathbf{a}, z)) c_{ij}(\mathbf{a}, z) \right] + \beta \mathbb{E} \frac{1}{\sigma_\epsilon} \frac{\partial v_i(\mathbf{a}', z')}{\partial d_{ij} / d_{ij}}.\end{aligned}$$

To get a sense of things, vary the second term by wealth. . .

$$\frac{\partial (u'(c_{ij}(\mathbf{a}, z)) c_{ij}(\mathbf{a}, z))}{\partial \mathbf{a}} = u'(c_{ij}(\mathbf{a}, z)) \times \text{MPC}_{ij}(\mathbf{a}, z) \times \left[-\rho_{ij}(\mathbf{a}, z) + 1 \right],$$

where $\rho_{ij}(\mathbf{a}, z)$ is the Arrow-Pratt measure of relative risk aversion.

With CRRA, if risk aversion > 1 , then poor, high marginal utility households (who are also high MPC households) are *more elastic relative* to rich households on the extensive margin.

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Bilateral Trade Elasticities: German Example

