Discrete Choice, Complete Markets, and Equilibrium

Simon Mongey Michael E. Waugh
FRB Minneapolis and NBER

June 30, 2023

The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

The Efficiency Properties of Discrete Choice Models?

We are interested in the efficiency properties of models where households have preferences

$$u(c_j) + \xi_j$$

over a finite set of goods J and households can only consume one good type.

Models of this nature have proven to be useful in many applications...

- consumer demand — McFadden (1974), Berry, Levinsohn, and Pakes (1995)
- location and migration — Redding and Rossi-Hansberg (2017)
- amenity driven occupational choice / labor supply — Berger, Herkenhoff, and Mongey (2022)

However, we don't know much (at least Simon and I) if welfare theorems hold in these economies, how one would even think about the problem, does it matter?

What We Find. . .

1. The standard allocations arising out of these models are inefficient.

- The key issue is market incompleteness. Households would like insurance against "choice risk".

2. We characterize the complete markets allocation, planner's problem, and provide welfare theorems.

- The novelty is figuring out how households choose the good when they have insurance — the result is the goods choice is **not** max over utility as in the incomplete markets problem.
- Widely used case of log preferences $\Rightarrow$ incomplete markets coincides with complete markets!

3. The implications of these results for settings when firms have market power

- Complete markets / efficiency on the household side effects the elasticity of demand
- Two effects: (i) overall less elastic (bad), (ii) but more elastic for highest price firms (good)

Model: Households

M types of households with names $i = \{1,\ 2\ldots, M\}$ and mass μ^i households of each type

What does each household do? Households work.

They supply n^i units of labor in a competitive labor market to firms producing differentiated consumption goods.

Model: Households

M types of households with names $i = \{1,\ 2\ldots, M\}$ and mass μ^i households of each type

What does each household do? Households work. Households consume.

They face J differentiated goods with names $j = \{1,\ 2\ldots, J\}$ and ...

- Households receive random realization of taste shocks

$$\boldsymbol{\xi} = (\xi_1, \ldots, \xi_j, \ldots, \xi_J) \quad \text{with PDF} \quad g(\boldsymbol{\xi}),$$

  that are independently distributed in the population.

- Households can chose only one good to consume. Utility conditional on choosing good j:

$$u(c_j) + \xi_j.$$

Model: Households

M types of households with names $i = \{1,\ 2\ldots, M\}$ and mass μ^i households of each type

What does each household do? Households work. Households consume.

They face J differentiated goods with names $j = \{1,\ 2\ldots, J\}$ and . . .

- Households receive random realization of taste shocks

$$\boldsymbol{\xi} = (\xi_1, \ldots, \xi_j, \ldots, \xi_J) \quad \text{with PDF} \quad g(\boldsymbol{\xi}),$$

  that are independently distributed in the population.

- Households can chose only one good to consume. Utility conditional on choosing good j:

$$u(c_j) + \xi_j.$$

We execute things generally —just need u and g to be well behaved.

Model: Production

Competitive firms (we relax later) produce variety j with:

$$y_j = z_j N_j,$$

where z_j is TFP; N_j are total units of labor supplied by households.

This structure leads to the following prices that households face

$$p_j = \frac{W}{z_j},$$

where W is the wage rate.

Outline

That's the environment. . .

Next steps:

- Characterize the "standard" (incomplete markets) equilibrium and show standard results.
- Show there are feasible allocations that dominate the standard allocation.
- The model with complete markets, the planning problem, and welfare theorems.
- Log preferences
- What happens when we relax competitive product markets?
- The Welfare effects of price changes.

The Households' Problem

We setup the households' problem where they formulate plans that map a realization of $\boldsymbol{\xi}$ into a commodity choice and consumption quantity.

Problem of a household of type i

$$\max_{c^i_j(\boldsymbol{\xi}),\ x^i_j(\boldsymbol{\xi})} \int_{\boldsymbol{\xi}} \sum_j x^i_j(\boldsymbol{\xi})\Big[u\Big(c^i_j(\boldsymbol{\xi})\Big) + \xi_j\Big] g(\boldsymbol{\xi}) d\boldsymbol{\xi}\ , \qquad \text{subject to:}$$

$$\big[\lambda^i(\boldsymbol{\xi})\big]: \qquad \sum_j x^i_j(\boldsymbol{\xi}) p_j c^i_j(\boldsymbol{\xi}) \leq\ Wn^i \quad \forall \boldsymbol{\xi},$$

where

- $x^i_j(\boldsymbol{\xi})$ is an indicator function mapping $\boldsymbol{\xi}$ into a one if j is chosen and zero otherwise;
- $c^i_j(\boldsymbol{\xi})$ maps $\boldsymbol{\xi}$ into the quantity consumption of commodity j, if chosen.
- $\lambda^i(\boldsymbol{\xi})$ is the multiplier on the household's budget constraint for each $\boldsymbol{\xi}$.

Characterizing the Household's Problem

Fix a $\boldsymbol{\xi}$ and then start making comparisons across different options

$$\left[u(c_1^i(\boldsymbol{\xi})) + \xi_1\right] g(\boldsymbol{\xi}) - \lambda^i(\boldsymbol{\xi}) p_1 c_1(\boldsymbol{\xi}) \quad \text{vs.}$$

$$\left[u(c_2^i(\boldsymbol{\xi})) + \xi_2\right] g(\boldsymbol{\xi}) - \lambda^i(\boldsymbol{\xi}) p_2 c_2(\boldsymbol{\xi}) \quad \ldots$$

Fix a $\boldsymbol{\xi}$ and then start making comparisons across different options

$$\Big[u(c_1^i(\boldsymbol{\xi})) + \xi_1\Big] g(\boldsymbol{\xi}) - \lambda^i(\boldsymbol{\xi}) Wn^i \quad \text{vs.}$$

$$\Big[u(c_2^i(\boldsymbol{\xi})) + \xi_2\Big] g(\boldsymbol{\xi}) - \lambda^i(\boldsymbol{\xi}) Wn^i \quad \ldots$$

Characterizing the Household's Problem

Fix a $\boldsymbol{\xi}$ and then start making comparisons across different options

$$\left[u(c_1^i(\boldsymbol{\xi})) + \xi_1\right] \quad \text{vs.}$$

$$\left[u(c_2^i(\boldsymbol{\xi})) + \xi_2\right] \quad \ldots$$

Characterizing the Household's Problem

The optimal $x_j^i(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i(\boldsymbol{\xi})) + \xi_j \quad \geq \quad \max_k \left\{ \; u(c_k^i(\boldsymbol{\xi})) + \xi_k \; \right\} \\ 0, & \text{otherwise} \end{cases}$$

Characterizing the Household's Problem

The optimal $x_j^i(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i(\boldsymbol{\xi})) + \xi_j \quad \geq \quad \max_k \left\{ \; u(c_k^i(\boldsymbol{\xi})) + \xi_k \; \right\} \\ 0, & \quad \text{otherwise} \end{cases}$$

Consumption satisfies

$$\frac{u'(c_j^i(\boldsymbol{\xi}))}{p_j} = \lambda_j^i(\boldsymbol{\xi}) \quad , \quad c_j^i(\boldsymbol{\xi}) = \frac{Wn^i}{p_j}$$

Characterizing the Household's Problem

The optimal $x_j^i(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i \quad) + \xi_j \;\; \geq \;\; \max_k \left\{ \; u(c_k^i \quad) + \xi_k \; \right\} \\ 0, & \text{otherwise} \end{cases}$$

Consumption satisfies

$$\frac{u'(c_j^i \quad)}{p_j} = \lambda_j^i \qquad , \quad c_j^i \quad = \frac{Wn^i}{p_j}$$

Characterizing the Household's Problem

The optimal $x_j^i(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i \quad) + \xi_j \;\; \geq \;\; \max_k \left\{ \; u(c_k^i \quad) + \xi_k \; \right\} \\ 0, & \text{otherwise} \end{cases}$$

Consumption satisfies

$$\frac{u'(c_j^i \quad)}{p_j} = \lambda_j^i \qquad , \quad c_j^i \quad = \frac{Wn^i}{p_j}$$

Assume the $\boldsymbol{\xi}$'s are distributed Type 1 Extreme Value with parameter η and our $x_j(\boldsymbol{\xi}) \Rightarrow$

$$\rho_j^i = \exp\left(\frac{u(c_j^i)}{\eta}\right) \Big/ \sum_k \exp\left(\frac{u(c_k^i)}{\eta}\right)$$

which is the mass of i Households choosing j. Let's call this the *standard allocation*.

Could Households Do Better?

Everything looks good . . . but marginal utility (adjusted for prices) is **not** equated across events (ξ)

$$\frac{u'(c_j^i)}{p_j} \neq \frac{u'(c_k^i)}{p_k}$$

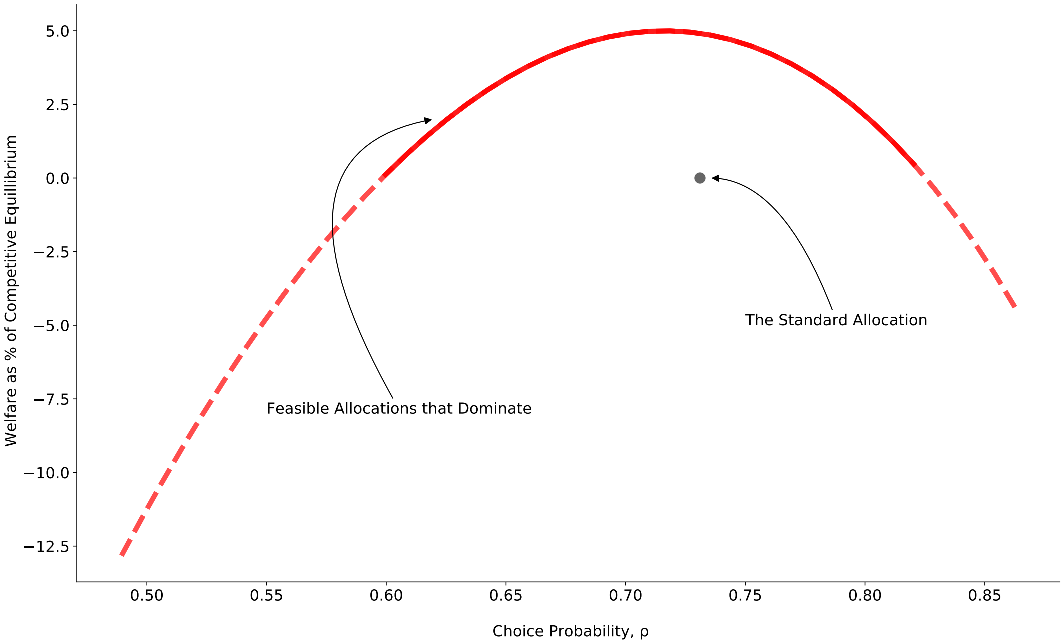
which suggests a failure of risk sharing.

Does this matter? Consider an alternative allocation where we

- Fix ρ_j^i's, then (i) exogenously impose a risk-sharing-like rule from Backus and Smith (1993), (ii) back out the levels of c_j^i's to ensure feasibility.
- These allocations are feasible, but not what arises in the standard allocation.

Next slide computes welfare in this alternative allocation and compares it to the standard allocation.

Could Households Do Better? Yes.



The Complete Markets Problem I

Same environment — but now households can purchase actuarially fair insurance. The problem:

$$\max_{a^i(\boldsymbol{\xi}), c^i_j(\boldsymbol{\xi}), x^i_j(\boldsymbol{\xi})} \int_{\boldsymbol{\xi}} \sum_j x^i_j(\boldsymbol{\xi}) \Big[u(c^i_j(\boldsymbol{\xi})) + \xi_j \Big] g(\boldsymbol{\xi}) d\boldsymbol{\xi}, \qquad \text{subject to:}$$

$$\sum_j x^i_j(\boldsymbol{\xi}) p_j c^i_j(\boldsymbol{\xi}) \leq \; W n^i + a^i(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi}$$

$$\int_{\boldsymbol{\xi}} q(\boldsymbol{\xi}) a^i(\boldsymbol{\xi}) d\boldsymbol{\xi} = 0$$

where the new notation is

- $q(\boldsymbol{\xi})$ is the state price for event $\boldsymbol{\xi}$,
- $a^i(\boldsymbol{\xi})$ are contingent claims that payout in event $\boldsymbol{\xi}$, zero otherwise.

The Complete Markets Problem II

We can rewrite this in "lifetime budget constraint form"

$$\max_{a^i(\boldsymbol{\xi}), c^i_j(\boldsymbol{\xi}), x^i_j(\boldsymbol{\xi})} \int_{\boldsymbol{\xi}} \sum_j x^i_j(\boldsymbol{\xi}) \Big[u(c^i_j(\boldsymbol{\xi})) + \xi_j \Big] g(\boldsymbol{\xi}) d\boldsymbol{\xi}, \qquad \text{subject to:}$$

$$[\lambda^i] : \qquad \int_{\boldsymbol{\xi}} q(\boldsymbol{\xi}) \Big[Wn^i - \sum_j x^i_j(\boldsymbol{\xi}) p_j c^i_j(\boldsymbol{\xi}) \Big] d\boldsymbol{\xi} = 0$$

and now there is one constraint with multiplier λ^i.

- This constraint is *the* distinguishing feature between CE ($\boldsymbol{\xi}$-by-$\boldsymbol{\xi}$ constraints) and the complete markets problem where all risk is consolidated.

Solving the Households Problem in Complete Markets

Same idea: fix a $\boldsymbol{\xi}$ and then start making comparisons across different options

$$\Big[u(c_1^i(\boldsymbol{\xi})) + \xi_1\Big] g(\boldsymbol{\xi}) - \lambda^i q(\boldsymbol{\xi}) p_1 c_1(\boldsymbol{\xi}) \quad \text{vs.}$$

$$\Big[u(c_2^i(\boldsymbol{\xi})) + \xi_2\Big] g(\boldsymbol{\xi}) - \lambda^i q(\boldsymbol{\xi}) p_2 c_2(\boldsymbol{\xi}) \quad \dots$$

Solving the Households Problem in Complete Markets

Same idea: fix a $\boldsymbol{\xi}$ and then start making comparisons across different options

Actuarially fair state prices $q(\boldsymbol{\xi}) = g(\boldsymbol{\xi}) \quad \Rightarrow$

$$\left[u(c_1^i(\boldsymbol{\xi})) + \xi_1 - \lambda^i p_1 c_1(\boldsymbol{\xi})\right] \quad \text{vs.} \quad \left[u(c_2^i(\boldsymbol{\xi})) + \xi_2 - \lambda^i p_2 c_2(\boldsymbol{\xi})\right] \quad \ldots$$

Solving the Households Problem in Complete Markets

The optimal $x^i_j(\boldsymbol{\xi})$ takes the form

$$x^i_j(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if} \;\; u(c^i_j(\boldsymbol{\xi})) + \xi_j - \lambda^i p_j c^i_j(\boldsymbol{\xi}) \;\; \geq \;\; \max_k \left\{ \; u(c^i_k(\boldsymbol{\xi})) + \xi_k - \lambda^i p_k c^i_k(\boldsymbol{\xi}) \; \right\} \\ 0, & \;\text{otherwise} \end{cases}$$

Solving the Households Problem in Complete Markets

The optimal $x_j^i(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i(\boldsymbol{\xi})) + \xi_j - \lambda^i p_j c_j^i(\boldsymbol{\xi}) \;\; \geq \;\; \max_k \left\{ \; u(c_k^i(\boldsymbol{\xi})) + \xi_k - \lambda^i p_k c_k^i(\boldsymbol{\xi}) \; \right\} \\ 0, & \text{otherwise} \end{cases}$$

Consumption satisfies

$$\frac{u'(c_j^i(\boldsymbol{\xi}))}{p_j} = \lambda^i$$

Solving the Households Problem in Complete Markets

The optimal $x_j^i(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i \quad) + \xi_j - \lambda^i p_j c_j^i \;\geq\; \max_k \left\{ \; u(c_k^i \quad) + \xi_k - \lambda^i p_k c_k^i \quad \right\} \\ 0, & \text{otherwise} \end{cases}$$

Consumption satisfies

$$\frac{u'(c_j^i \quad)}{p_j} = \lambda^i$$

Solving the Households Problem in Complete Markets

The optimal $x_j^i(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i \quad) + \xi_j - \lambda^i p_j c_j^i \;\; \geq \;\; \max_k \left\{ u(c_k^i \quad) + \xi_k - \lambda^i p_k c_k^i \quad \right\} \\ 0, & \text{otherwise} \end{cases}$$

Consumption satisfies

$$\frac{u'(c_j^i \quad)}{p_j} = \lambda^i$$

Assume the $\boldsymbol{\xi}$'s are distributed Type 1 Extreme Value with parameter η and our $x_j(\boldsymbol{\xi}) \Rightarrow$

$$\rho_j^i = \exp\left(\frac{u(c_j^i) - \lambda^i p_j c_j^i}{\eta}\right) \Bigg/ \sum_k \exp\left(\frac{u(c_k^i) - \lambda^i p_k c_k^i}{\eta}\right)$$

which is **not** what arises in the standard allocation.

Solving the Households Problem in Complete Markets

The optimal $x_j^i(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i \quad) + \xi_j - \lambda^i p_j c_j^i \;\; \geq \;\; \max_k \left\{ u(c_k^i \quad) + \xi_k - \lambda^i p_k c_k^i \quad \right\} \\ 0, & \text{otherwise} \end{cases}$$

Consumption satisfies

$$\frac{u'(c_j^i \quad)}{p_j} = \lambda^i$$

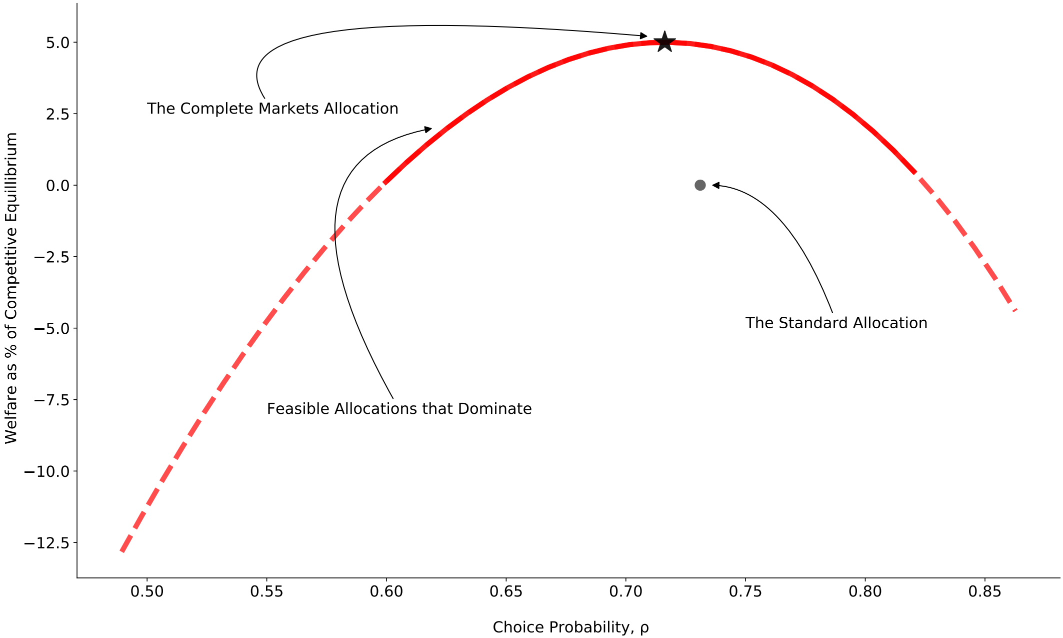
Both the quantity and choice differ from the standard allocation.

- Ratios of marginal utilities equal relative prices — now we have a risk-sharing-like condition.
- The form that $x_j^i(\boldsymbol{\xi})$ is novel / the unique contribution of the paper ...

  Incomplete markets says — "chose j with highest utility"

  Complete markets says — "chose j with highest utility **net of the cost**"

The Complete Markets Allocation



The Planning Problem

Same environment — but now a Social Planner can directly choose the allocation. The problem:

$$\max_{n_j,\, c_j^i(\boldsymbol{\xi}),\, x_j^i(\boldsymbol{\xi})} \sum_i \mu^i \theta^i \int_{\boldsymbol{\xi}} \sum_j x_j^i(\boldsymbol{\xi}) \Big[u(c_j^i(\boldsymbol{\xi})) + \xi_j \Big] g(\boldsymbol{\xi}) d\boldsymbol{\xi}, \qquad \text{subject to:}$$

$$[\Lambda_j]: \qquad \sum_i \mu^i \int_{\boldsymbol{\xi}} x_j^i(\boldsymbol{\xi}) c_j^i(\boldsymbol{\xi}) g(\boldsymbol{\xi}) d\boldsymbol{\xi} \;\leq\; z_j N_j \quad \forall j = 1, \ldots, J$$

$$[\Lambda_n]: \qquad \sum_j N_j \leq \sum_i \mu^i \; n^i$$

where the new notation is

- θ^i is the Pareto weight for Households of type i.
- Λ_j and Λ_n are multipliers on goods and labor resource constraints

Characterizing the Planning Problem

The optimal $x_j(\boldsymbol{\xi})$ takes the form

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; \theta^i \left[u(c_j^i) + \xi_j\right] - \Lambda_j c_j \quad \geq \quad \max_k \left\{ \; \theta^i \left[u(c_k^i) + \xi_k\right] - \Lambda_k c_k \; \right\} \\ 0, & \text{otherwise} \end{cases}$$

Consumption satisfies

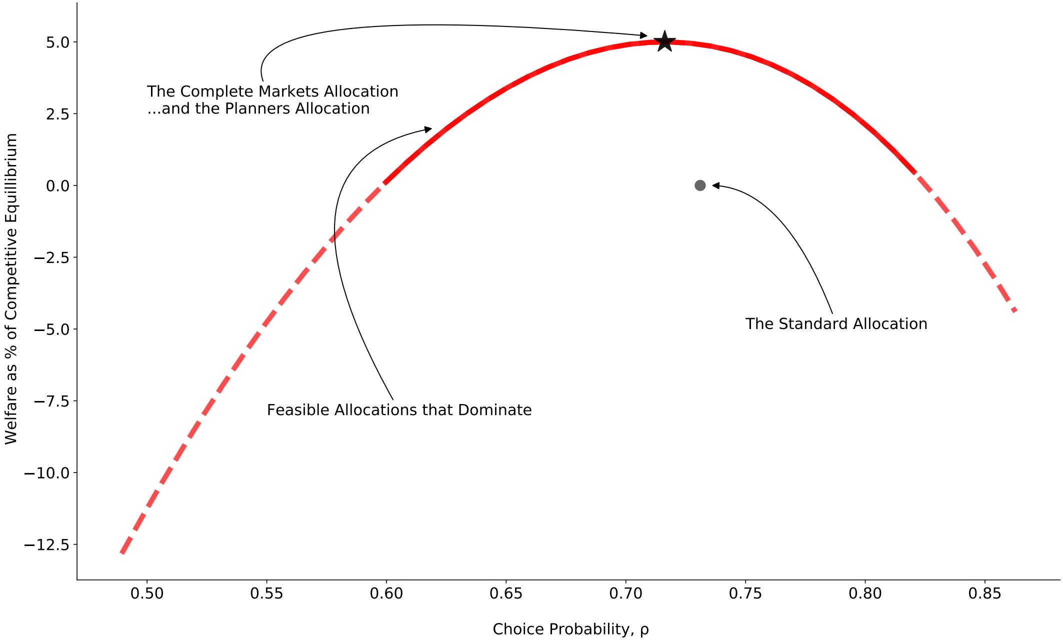
$$\theta^i u'(c_j^i) = \Lambda_j \quad , \quad \Lambda_j = \frac{\Lambda_n}{z_j} \qquad \text{Efficient allocation}$$

$$u'(c_j^i) = \lambda^i p_j \quad , \quad p_j = \frac{W}{z_j} \qquad \text{Complete markets}$$

Now we can start to see the equivalence between Planner and Complete Markets

- FOC on c_j^i + FOC on N_j $\Rightarrow$ Ratios of marginal utilities equal relative productivity.
- Form of $x_j^i(\boldsymbol{\xi})$ compares "social benefit" to "social cost" of consumption if choose good j.

The Planner's Allocation



Definition — Allocation

- *An allocation is product choice $x_j^i(\boldsymbol{\xi})$, and consumption $c_j^i(\boldsymbol{\xi})$, for all i, j, $\boldsymbol{\xi}$, and labor N_j for all j.*

Result 1 — First welfare theorem

- *There exists a vector of Pareto weights $\boldsymbol{\theta}$ such that the planner's allocation and competitive equilibrium complete markets allocation coincide*

Result 2 — Second welfare theorem

- *For any $\boldsymbol{\theta}$, there exists a set of budget neutral lump-sum transfers such that the planner's allocation is obtained in a competitive equilibrium with complete markets*

Result 3 — Arrow vouchers

- *The first and second welfare theorems hold under a restricted set of securities that pay off conditional* on purchasing good j
- Recall that $c_j^i(\boldsymbol{\xi})$ was independent of ξ conditional on j

log Preferences

Log $u(c)$ + Logit ξ (Type 1 Extreme Value) is a very common / important setting

- Anderson, De Palma, and Thisse (1992) use this to construct a CES representative consumer.
- Berger, Herkenhoff, and Mongey (2022) adapt to nested CES and labor markets.
- A core piece of quantitative spatial models — Redding and Rossi-Hansberg (2017).

Result 4 - *Under* log*, the first and second welfare theorem hold with incomplete markets.*

log Preferences

Log $u(c)$ + Logit $\boldsymbol{\xi}$ (Type 1 Extreme Value) is a very common / important setting

- Anderson, De Palma, and Thisse (1992) use this to construct a CES representative consumer.
- Berger, Herkenhoff, and Mongey (2022) adapt to nested CES and labor markets.
- A core piece of quantitative spatial models — Redding and Rossi-Hansberg (2017).

Result 4 - *Under* log*, the first and second welfare theorem hold with incomplete markets.*

The choice rule in complete markets / planner problem

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i) + \xi_j - u'(c_j^i)c_j^i \;\; \geq \;\; \max_k \left\{ \; u(c_k^i) + \xi_k - u'(c_k^i)c_k^i \; \right\} \\ 0, & \text{otherwise} \end{cases}$$

log Preferences

Log $u(c)$ + Logit $\boldsymbol{\xi}$ (Type 1 Extreme Value) is a very common / important setting

- Anderson, De Palma, and Thisse (1992) use this to construct a CES representative consumer.
- Berger, Herkenhoff, and Mongey (2022) adapt to nested CES and labor markets.
- A core piece of quantitative spatial models — Redding and Rossi-Hansberg (2017).

Result 4 - *Under* log*, the first and second welfare theorem hold with incomplete markets.*

The choice rule in complete markets / planner problem ... with log

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i) + \xi_j - 1 \quad \geq \quad \max_k \left\{ \; u(c_k^i) + \xi_k - 1 \; \right\} \\ 0, & \text{otherwise} \end{cases}$$

log Preferences

Log $u(c)$ + Logit $\boldsymbol{\xi}$ (Type 1 Extreme Value) is a very common / important setting

- Anderson, De Palma, and Thisse (1992) use this to construct a CES representative consumer.
- Berger, Herkenhoff, and Mongey (2022) adapt to nested CES and labor markets.
- A core piece of quantitative spatial models — Redding and Rossi-Hansberg (2017).

Result 4 - *Under* log*, the first and second welfare theorem hold with incomplete markets.*

which is the same as under incomplete markets

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i) + \xi_j \quad \geq \quad \max_k \left\{ \; u(c_k^i) + \xi_k \quad \right\} \\ 0, & \text{otherwise} \end{cases}$$

log Preferences

Log $u(c)$ + Logit $\boldsymbol{\xi}$ (Type 1 Extreme Value) is a very common / important setting

- Anderson, De Palma, and Thisse (1992) use this to construct a CES representative consumer.
- Berger, Herkenhoff, and Mongey (2022) adapt to nested CES and labor markets.
- A core piece of quantitative spatial models — Redding and Rossi-Hansberg (2017).

Result 4 - *Under* log*, the first and second welfare theorem hold with incomplete markets.*

which is the same as under incomplete markets

$$x_j^i(\boldsymbol{\xi}) = \begin{cases} 1, & \text{if } \; u(c_j^i) + \xi_j \quad \geq \quad \max_k \left\{ \; u(c_k^i) + \xi_k \quad \right\} \\ 0, & \text{otherwise} \end{cases}$$

And then marginal rates of substitution equal to ratios of prices

$$c_j^i = \frac{Wn^i}{p_j} \quad \Longrightarrow \quad \frac{u'(c_j^i)}{p_j} = \frac{u'(c_k^i)}{p_k} = \frac{1}{Wn^i} \quad \forall \; j, k$$

log Preferences

Log $u(c)$ + Logit ξ (Type 1 Extreme Value) is a very common / important setting

- Anderson, De Palma, and Thisse (1992) use this to construct a CES representative consumer.
- Berger, Herkenhoff, and Mongey (2022) adapt to nested CES and labor markets.
- A core piece of quantitative spatial models — Redding and Rossi-Hansberg (2017).

Result 4 - *Under* log, *the first and second welfare theorem hold with incomplete markets.*

Lot's of thoughts about this...

- Reminiscent of Cole and Obstfeld (1991) — novelty here is how things wash out in choice rule x_j^i.
- Not an issue about the distribution on ξ — this was my (wrong) conjecture for a while
- Open questions — Pushing on ADPT (1992) ...can closed form rep. agent be derived under:
    1. Log and arbitrary $G(\xi)$?
    2. Non-log and complete markets with logit $G(\xi)$?

Consumption vs. Productive Efficiency

We've shown that complete markets + competitive pricing yield allocative efficiency.

What happens if markets are complete but pricing is not competitive?

Special case: (i) CRRA $u(c)$ with parameter σ, (ii) Type 1 EV on $\boldsymbol{\xi}$ with parameter η, (iii) $n^i = \overline{n}$

Firms will price as a markup μ_j over marginal cost, what is their elasticity of demand?

Consumption vs. Productive Efficiency

We've shown that complete markets + competitive pricing yield allocative efficiency.

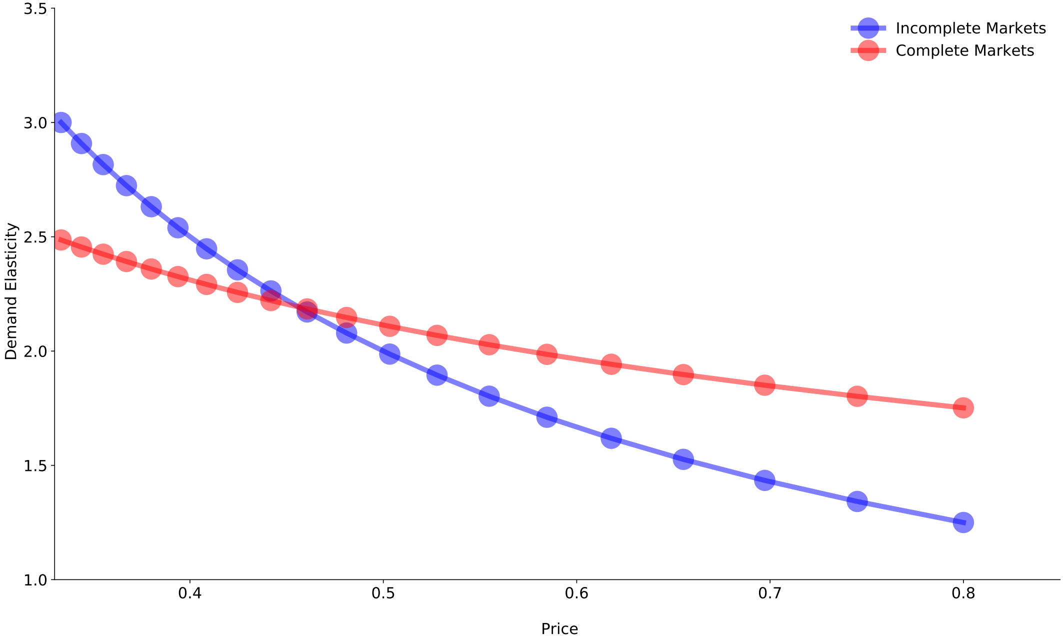
What happens if markets are complete but pricing is not competitive?

Special case: (i) CRRA $u(c)$ with parameter σ, (ii) Type 1 EV on $\boldsymbol{\xi}$ with parameter η, (iii) $n^i = \overline{n}$

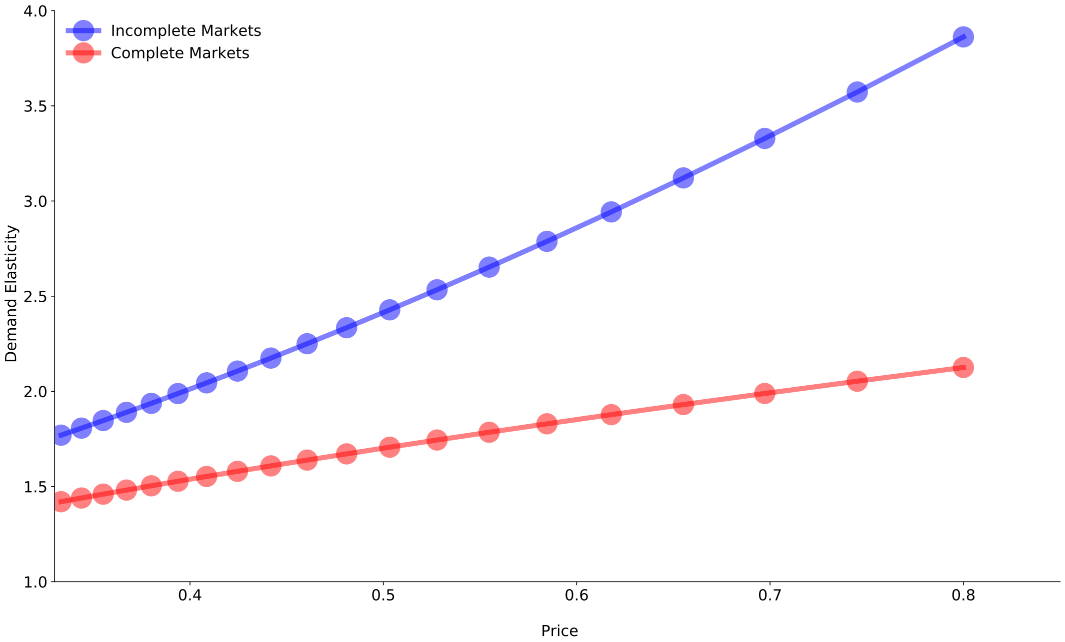
$$\text{Incomplete :} \qquad c_j = \frac{W\overline{n}}{p_j}\ ,\ \ \rho_j = \frac{\exp\{u(c_j)\eta\}}{\sum_k \exp\{u(c_k)\eta\}}\ ,\quad \varepsilon_j = 1 + \eta\left(\frac{p_j}{W\overline{n}}\right)^{\sigma-1}$$

$$\text{Complete :} \qquad c_j^{-\sigma} = \lambda p_j\ ,\ \ \rho_j = \frac{\exp\{u(c_j)\sigma\eta\}}{\sum_k \exp\{u(i_k)\sigma\eta\}}\ ,\ \ \varepsilon_j = \frac{1}{\sigma} + \eta\left(\frac{p_j/\Delta_j}{W\overline{n}}\right)^{\sigma-1}\ ,\ \ \Delta_j = \frac{p_j^{\frac{\sigma-1}{\sigma}}}{\sum_k \rho_k p_k^{\frac{\sigma-1}{\sigma}}}$$

Consumption in Incomplete and Complete Markets



Demand Elasticities in Incomplete and Complete Markets



Consumption Inefficiency $\Rightarrow$ Production Inefficiency?

Two off-setting forces

1. Efficient consumption moves resources to make consumption less sensitive to pricing $\rightarrow$ Higher μ.
2. Especially so at high p_j, low z_j firms $\rightarrow$ Higher μ_j at low z_j firms.

Not clear yet which wins.

But there is an interesting idea here: market incompleteness on the household side leads to misallocation on the production side.

Welfare Effects of Price Changes

- **Price Theory 101** — Equivalent variation ψ

$$
\begin{aligned}
V(\boldsymbol{p}, y) &= \max_{\boldsymbol{c}} u(c_1, \ldots, c_J) \quad s.t. \quad \sum_j p_j c_j = y \\
V(\boldsymbol{p}, (1-\psi) y) &= V(\boldsymbol{p} + d\boldsymbol{p}, y) \\
\psi &= \sum_j \left(\frac{p_j c_j}{y} \right) d \log p_j
\end{aligned}
$$

Independent of the form of u and $G(\boldsymbol{\xi})$:

- **Complete markets**

$$\psi = \sum_j \left(\frac{\rho_j p_j c_j}{y} \right) d \log p_j$$

- **Incomplete markets**

$$\psi = \sum_j \left(\frac{\rho_j u'(c_j) / p_j}{\sum_k \rho_k u'(c_k) / p_k} \right) \left(\frac{p_j c_j}{y} \right) d \log p_j$$

- **Result:** *Discrete choice + incomplete markets $\Rightarrow$ standard welfare effects of price changes formulas are not empirically relevant ... except in the knife-edge case of* log

First Order Welfare Effects of Productivity Shocks

Independent of the form of u and $G(\xi)$:

- **Price Theory 101**

$$d\log U = \sum_j \frac{p_j c_j}{\sum_k p_k c_k} d\log z_j$$

- **Complete markets**

$$d\log U = \sum_j \frac{\rho_j p_j c_j}{\sum_k \rho_k p_k c_k} d\log z_j$$

- **Incomplete markets**

$$d\log U = \sum_j \frac{\lambda_j}{\sum_k \lambda_k} d\log z_j = \sum_j \frac{\rho_j u'(c_j)/p_j}{\sum_k \rho_k u'(c_k)/p_k} d\log z_j$$

- **Result** — *Hulten's Theorem 'like' results also hold nicely with complete markets (efficient economy), but fail with incomplete markets*

- **Note** - This is assuming efficient production. What happens with inefficient production?

Conclusion

We came at this problem through the lens of our other work in Mongey and Waugh (2023):

- Are allocations efficient in discrete choice models? **NO**
- Is their a role for insurance? **YES**

These answers then. . .

- Motivate the importance of partial insurance (Bewley (1979) etc.) in the context of discrete choices like products, location, sector, etc.
- Deliver a new idea as to where markups and misallocation arise from — not "technologically determined" through preferences, but market incompleteness on the household side.

References I

Anderson, S. P., A. De Palma, and J.-F. Thisse (1992): *Discrete choice theory of product differentiation*, MIT press.

Backus, D. K. and G. W. Smith (1993): "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, 35, 297 – 316.

Berger, D., K. Herkenhoff, and S. Mongey (2022): "Labor market power," *American Economic Review*, 112, 1147–93.

Berry, S., J. Levinsohn, and A. Pakes (1995): "Automobile prices in market equilibrium," *Econometrica: Journal of the Econometric Society*, 841–890.

Bewley, T. (1979): "The optimum quantity of money," Tech. rep., Discussion Paper.

Cole, H. L. and M. Obstfeld (1991): "Commodity trade and international risk sharing: How much do financial markets matter?" *Journal of monetary economics*, 28, 3–24.

McFadden, D. (1974): ",Conditional logit analysis of qualitative choice behavior," in *Frontiers in Econometrics*, Academic Press New York, NY, USA.

Mongey, S. and M. Waugh (2023): "Pricing Inequality," .

Redding, S. J. and E. Rossi-Hansberg (2017): "Quantitative spatial economics," *Annual Review of Economics*, 9, 21–58.