Redistributing the Gains From Trade Through Progressive Taxation

Spencer G. Lyon
New York University

Michael E. Waugh
New York University and NBER

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ABSTRACT

Should a nation’s tax system become more progressive as it opens to trade? Does opening to trade change the benefits of a progressive tax system? We answer these question within a standard incomplete markets model with frictional labor markets and Ricardian trade. Consistent with empirical evidence, adverse shocks to comparative advantage lead to labor income losses for import-competition-exposed workers; with incomplete markets, these workers are imperfectly insured and experience welfare losses. A progressive tax system is valuable, as it substitutes for imperfect insurance and redistributes the gains from trade. However, it also reduces the incentives for labor to reallocate away from comparatively disadvantaged locations. We find that optimal progressivity should increase with openness to trade with a ten percentage point increase in openness necessitating a five percentage point increase in marginal tax rates for those at the top of the income distribution.

Email: spencer.lyon@stern.nyu.edu, mwaugh@stern.nyu.edu. We would like to thank our discussant Oleg Itskhoki, Gordon Hanson, Stephen Redding, Referees, and participants at the NBER Trade and Labor Markets Conference and Vanderbilt University for valuable comments and suggestions.
1. Introduction

There are many concerns about the forces of globalization—that the losses from trade are large; that there are insufficient mechanisms to insure against these losses; that globalization simply propagates existing inequality. The standard response to these concerns is helpful in theory: that there exists a Pareto improving transfer scheme that can compensate the losers from trade, yet still preserve the gains for the winners. In practice, this response is less helpful, given the limited mechanisms and incentive problems that policy makers face in implementing any kind of transfer scheme.

Evidence suggests that these concerns about the forces of globalization are warranted. Autor, Dorn, and Hanson (2013) show that exposure to Chinese import competition has led to losses in labor income and reductions in labor force participation for import-competition-exposed workers in the United States Krishna and Senses (2014) show that increases in import penetration are associated with increases in labor income risk. Pavcnik (2017) surveys the growing body of evidence regarding trade’s affect on earnings and employment opportunities. Given that risk sharing is often found to be incomplete (see, e.g., Cochrane (1991), Attanasio and Davis (1996)), this suggests that the labor market consequences of trade lead to welfare losses.

Policy need not be silent to these concerns. One way to mitigate and insure against these losses is via the tax system. That is, the government could use a progressive tax system to provide social insurance that helps transfer resources from the winners from trade to the losers. This paper evaluates this possibility by measuring both the optimal degree of tax progressivity and the gains from progressivity as an economy opens to trade.

We implement these ideas by building off of our parallel work in Lyon and Waugh (2018). In this work, we develop an open-economy, standard incomplete markets model with frictional labor markets. The open-economy aspect of our model builds on existing trade theory by developing a dynamic Ricardian model of trade and frictional labor markets. As in the model of Eaton and Kortum (2002), there is a continuum of goods with competitive producers who are heterogenous in productivity; comparative advantage determines the pattern of trade. As in Lucas and Prescott (1974) the labor market is frictional and labor can only move across different goods producing markets (within a country) after paying some cost.

As in the standard incomplete markets model (Huggett (1993), Aiyagari (1994)) households can self-insure by accumulating a non-state contingent asset. In addition, we endow households with several additional margins to mitigate labor income risk. Specifically, households can opt out of the labor force and enjoy leisure and/or migrate to better labor markets.

1Varian (1980) and Eaton and Rosen (1980) are early contributions showing how a progressive tax system provides social insurance.
These mechanisms give rise to an optimal degree of tax progressivity which balances the benefits of providing social insurance versus the costs of reducing labor supply and migration. Social insurance is valuable in our model because frictional labor markets and market incompleteness imply that households are exposed to idiosyncratic income risk, some of which is trade related. A progressive tax system provides a mechanism to substitute for imperfect insurance against trade and non-trade related labor income risk.

On the other hand, a progressive tax system comes with the cost of reducing labor supply and migration. As emphasized in the optimal taxation literature, labor income taxes reduce the incentive to work and, thus, shrinks the size of the pie available for redistribution. Our model highlights a new, second cost of a progressive tax system—it reduces the incentives for labor to migrate. A progressivity shrinks the gains to households from moving away from low-productivity to high-productivity places. Yet achieving allocative efficiency requires the continual movement of households from low- to high-productivity places. Thus, a progressive tax system contributes to the misallocation of households across space.

How the optimal degree of progressivity changes with increased openness to trade depends on the relative change in the benefits and costs. As an economy opens to trade, trade changes the labor market consequences of various shocks, which, in turn, increases the benefits of social insurance. However, the quantitative issue is how the costs of redistribution change with increased openness to trade. Are reductions in labor supply more costly as we open to trade? Are reductions in migration more costly? We quantitatively answer these question in the following way.

First, we place a restriction on the tax instruments the government has access to. In particular, we model the government as using a log-linear labor-income tax and transfer scheme to redistribute resources. In particular, we closely follow the approach of Benabou (2002), Conesa and Krueger (2006), and Heathcote, Storesletten, and Violante (2014) who parameterize the progressivity of net-taxes directly. As in that work, we take a stand on the social welfare function and we measure the optimal degree of progressivity as the progressivity parameter that maximizes social welfare.

Second, we calibrate the parameters of the model by having the model replicate key aggregate and cross-sectional moments of the US economy. Along with relatively standard parameter values for preferences and technologies, we insure that the model replicates aggregate trade exposure, internal migration rates, labor force participation rates, and the amount of indebtedness of households in the US economy. A key issue is always the elasticity of labor supply and we follow the work of Rogerson (1988) and more closely Chang and Kim (2007). In particular,

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2Heathcote, Storesletten, and Violante (2014) show that this functional form provides a good approximation of the actual tax and transfer scheme in the US data. Guner, Kaygusuz, and Ventura (2014) provide an exploration of this and alternative tax functions.
the micro-labor supply elasticity is low (zero in our case) but can diverge from the aggregate labor supply elasticity due to movements in and out of the labor force, i.e., the extensive margin. This formulation is broadly consistent with the evidence in Keane (2011).

We use the calibrated model as laboratory to answer several questions.

**How does the optimal tax policy change with increased openness to trade?** We find that the tax system should become more progressive with increased openness to trade. Starting from an economy with an import share of GDP of 10 percent (consistent with the US experience in the 1990s) and moving to an an import share of GDP at 20 percent, optimal progressivity increases with large increases in marginal tax rates for those at the top of the income distribution and decreases for those at the bottom. More generally, we find that optimal policy dictates that a ten percentage point increase in the import share of GDP necessitate a five percentage point increase in marginal tax rates for those in the 90 percentile of the income distribution.

The reason for this finding is that the output costs of progressive taxation are essentially constant across different levels of openness, but the benefits increase with openness. In particular, increased openness increases uninsurable income risk consistent with the findings of Krishna and Senses (2014). And this motivates the increased provision of social insurance. Thus the cost-benefit analysis tilts towards becoming more progressive as the economy opens up.

With that said, the welfare gains from a move to the optimal policy are modest. Only at large levels of trade exposure (at least for the US, but comparable to economies such as Canada and Mexico) do we find large welfare gains associated with a move towards optimal policy.

**How does openness change the benefits of a progressive tax system?** We find that a progressive tax system becomes systematically more beneficial as an economy opens up to trade. We illustrate this point by measuring the welfare gains of current tax policy relative to a flat tax system and how the gains change with increased openness to trade. In other words, this exercise measures the how the benefits of social insurance change with increased exposure to trade.

A move to a flat tax system—at current levels of openness—would lead to a three-quarters of a percent decrease in welfare. Moreover, the benefits of a progressive tax system become substantially larger as the economy becomes more open. This shows that a progressive tax system is an important tool towards enhancing welfare as an economy opens to trade.

This result also implies that the progressivity in the tax system increases the gains from trade relative to the calculations typically performed (see, e.g, Arkolakis, Costinot, and Rodríguez-Clare (2012)). For example, we find that the gains from trade are 25 percent larger under a progressive tax system than a flat system.

**How does a progressive tax system compare to import tariffs?** We study the optimal policy mix between a progressive tax system and an import tariff. This comparison is of interest for
two reasons. First, in the United States, policy makers are taking a second look at anti-trade commercial policy as a means to correct various imbalances and harms associated with trade.

Second, it has been argued that import tariffs and a progressive tax system may provide similar social insurance roles. Eaton and Grossman (1985) show that a motive for anti-trade commercial policy (i.e., a tariff) is to provide social insurance to the losers from trade (see, e.g., Corden (1974) and Baldwin (1982), as well). More starkly, Newbery and Stiglitz (1984) provide an example, in a setting with risk and incomplete markets, of free trade being Pareto inferior to autarky.³

To answer this final question, we measure the optimal policy mix between tax progressivity and import tariffs. In no cases do we find that a tariff is welfare-improving. The optimal mix is a zero tariff and a more progressive tax system as the economy becomes more exposed to trade.⁴

**Related Literature.** Conceptually, the ideas in this paper are closely related Rodrik (1997, 1998) and Epifani and Gancia (2009). Rodrik (1998) establishes a robust relationship between the size of a nation’s government and the extent to which it is open. An interpretation of this result is that nations use government spending to provide social insurance against the ills of globalization. With that said, there is a disconnect between our work and Rodrik’s (1998) evidence. In particular, we hold government expenditures fixed and households do not value them. However, our broader message—the increasing importance of government-provided social insurance with openness to trade—is very much consistent with Rodrik’s (1998) interpretation of the data and the related arguments in Rodrik (1997).

At a mechanical level, this paper is closely related to the quantitative studies of Conesa and Krueger (2006) and Heathcote, Storesletten, and Violante (2014), who study the optimal progressivity of the US tax scheme in heterogeneous agent, incomplete market models. As discussed above, we closely follow their and Benabou’s (2002) approach to parameterizing the tax and transfer scheme and focus on the tension between social insurance and economic efficiency. There are two distinguishing features of our paper. First, we highlight a new and distinct tension between social insurance and the misallocation of households across space. Second, we focus on how optimal progressivity changes as an economy becomes more open to trade.

The cost of following the quantitative literature is that we restrict the class of instruments the social planner has access to. Thus our analysis leaves open questions if alternative tax instruments may perform better then a non-linear labor income tax. For example, Dixit and Norman (1986) demonstrate how how commodity taxation can redistribute the gains from trade in a

³We take the reason that insurance markets are missing as given; Dixit (1987, 1989a,b) shows how these conclusion depend upon the modeling as to why these markets are missing.

⁴Relative to Eaton and Grossman (1985) or Newbery and Stiglitz (1984), we speculate that in their settings entertain alternative policy instruments and do not endow households with partial insurance opportunities. Moreover, one caveat to this analysis is that the tariff policy we consider is stark—one uniform tariff for all imported goods.
Pareto improving way. Our economy is much richer with desires for social insurance and partial factor mobility, however, exploring alternative instruments is an important direction for future work.

With regard to open economy issues, Spector (2001) and Antràs, De Gortari, and Itskhoki (2016) are closely related. Both papers focus on a static, open economy Mirrlees (1971) framework and study the welfare consequences of trade-induced inequality and its interaction with redistributive polices. In particular, Antràs, De Gortari, and Itskhoki (2016) study policies that take the same form as in our paper. The distinguishing feature of our work is that we focus on very different motives for redistributive taxation. In our model, the motive for progressive taxation is to provide social insurance and redistribute resources from the “lucky” towards the “unlucky.” This insurance motive is distinct from inequality aversion per se, as in Antràs, De Gortari, and Itskhoki (2016). This is the sense in which our paper builds most closely on the earlier work of Varian (1980), Eaton and Rosen (1980), and Mirrlees (1974).

Our modeling framework is related, but distinct from, an exciting and growing body work on trade and labor market dynamics (see, e.g., Kambourov (2009); Artuç, Chaudhuri, and McLaren (2010); Dix-Carneiro (2014); Caliendo, Dvorkin, and Parro (2015); Coşar, Guner, and Tybout (2016)). We depart from this literature by studying an economy in which households face labor income shocks, incomplete markets, and partial self-insurance. The cost of this departure is that we are unable to incorporate the geographic and sectoral detail found in this work (see, e.g., Caliendo, Dvorkin, and Parro (2015)) due to computational complexities. With that said, the benefits from this departure are important for several reasons.

First, the focus on a setting with incomplete markets opens up the door to a motive for government policy to provide social insurance and increased social insurance as the economy opens to trade. In other words, this allows us to study the normative implications of alternative policy schemes as an economy opens to trade. In contrast, the normative prescriptions are unclear (as well as unstudied) in previous work on trade and labor market dynamics.

Second, the migration motive in our model is for insurance. That is, households undertake costly moves to escape negative labor market conditions in one location and capture favorable labor market conditions in another location (see, e.g., Lagakos, Mobarak, and Waugh (2017) and references therein for the importance of this in the context of developing countries). This motive is distinct from the moving motive in the stationary equilibrium of Artuç, Chaudhuri, and McLaren (2010) and Caliendo, Dvorkin, and Parro (2015), which arises from shocks to preferences across locations, while income across locations is constant. Allowing for insurance-motivated moves is important as it creates a new, quantitatively important tension for policy.

\footnote{While a static framework, Galle, Rodríguez-Clare, and Yi (2017) is a closely related as well. Similarly motivated, they explicitly study the aggregate and distributional consequences of trade in an environment without ex-post transfers.}
makers to confront. That is a progressive tax system must balance the gains from social insurance versus the losses in allocative efficiency that come with less migration.

2. Model

Here, we describe a model of international trade with households facing incomplete markets and frictions to move across labor markets. The first subsection discusses the production structure; the second subsection discusses the government and the tax function; the third discusses the households.

Below, since we focus on the perspective of one country, country subscripts are omitted unless necessary. Similarly, because we focus on a stationary equilibrium, time subscripts are omitted unless necessary.

2.1. Production

The model has an intermediate-goods sector and a final good sector that aggregates the intermediate goods. Within a country, there is a continuum of intermediate goods indexed by \( \omega \in [0, 1] \). As in the Ricardian model of Dornbusch, Fischer, and Samuelson (1977) and Eaton and Kortum (2002), intermediate goods are not nationally differentiated. Thus, intermediate \( \omega \) produced in one country is a perfect substitute for the same intermediate \( \omega \) produced by another country.

Competitive firms produce intermediate goods with linear production technologies,

\[
q(\omega) = z(\omega)\ell, \tag{1}
\]

where \( z \) is the productivity level of firms and \( \ell \) is the number of efficiency units of labor. Intermediate goods productivity evolves stochastically according to an AR(1) process in logs

\[
\log z_{t+1} = \phi \log z_t + \epsilon_{t+1}, \tag{2}
\]

where \( \epsilon_{t+1} \) is distributed normally with mean zero and standard deviation \( \sigma_\epsilon \). The innovation \( \epsilon_{t+1} \) is independent across time, goods, and countries.

Firms producing variety \( \omega \) face competitive product and labor markets with households that supply labor elastically. Competition implies that a household choosing to work in market \( \omega \) earns the value of its marginal product of labor, which is the price of the good times the firm’s productivity \( z \).

Transporting intermediate goods across countries is costly. Specifically, firms face iceberg trade costs \( \tau \geq 1 \) when exporting their products. Abstracting from tariffs (which are discussed be-
low), this means that for a firm to deliver one unit of the intermediate good abroad, it must produce \( \tau \) units for shipment.

Intermediate goods are aggregated by a competitive final-goods producer who has a standard CES production function:

\[
Q = \left[ \int_0^1 q(\omega)^\rho d\omega \right]^{1/\rho},
\]

where \( q(\omega) \) is the quantity of individual intermediate goods \( \omega \) demanded by the final-goods firm, and \( \rho \) controls the elasticity of substitution across variety, which is \( \theta = \frac{1}{1-\rho} \).

2.2. Government

The government consumes resources \( G \), levies a labor income tax and transfer scheme, and taxes imports via tariffs.

**Government Consumption.** We model government consumption of resources \( G \) as pure waste. That is, there is no service flow to households from the government’s consumption. Motivating this modeling choice is the difficulty in disciplining the utility value of public goods (see, e.g., the discussion in Heathcote, Storesletten, and Violante (2014)). Thus, this choice reduces the number of free parameters we must take a stand on. The cost of this choice is that we abstract from a policy instrument—public goods provision—that also provides social insurance. And we lose the ability to speak to the evidence in Rodrik (1998) and the response of local public good provision in Feler and Senses (2017).

**Labor Income Tax and Transfer Scheme.** As in Benabou (2002) and Heathcote, Storesletten, and Violante (2014), we assume that net tax revenues are of the following parametric class:

\[
T(w) = w - \delta w^{1-\tau_p},
\]

where net tax revenues are \( T(w) \) and \( w \) is labor income. There are two parameters in (4). The \( \delta \) parameter determines the average rate and is chosen by the government such that its budget is balanced. The parameter \( \tau_p \) directly controls the progressivity of the tax scheme. This is the policy parameter of interest.

Heathcote, Storesletten, and Violante (2014) describe several ways to see how \( \tau_p \) determines the progressivity. The most straightforward way is to note that \( 1 - \tau_p \) equals one minus the marginal tax rate relative to one minus the average tax rate:

\[
1 - \tau_p = \frac{1 - T'(w)}{1 - T(w)/w}.
\]
Thus, when $\tau_p$ equals zero, marginal rates equal average rates—i.e., the tax system is neither regressive nor progressive and is deemed “flat.” In contrast, when $\tau_p$ is greater than zero, marginal rates $T'(w)$ exceed average rates, $T(w)/w$ and the tax system is deemed “progressive.”

In reality, the tax system is far more complex. We follow previous work by thinking about (4) vis-à-vis the data and the model as a scheme which encompass both the myriad of taxes and the transfers. That is take-home pay in the model reflects pregovernment income minus taxes plus transfers. Heathcote, Storesletten, and Violante (2014); Guner, Kaygusuz, and Ventura (2014); Antrás, De Gortari, and Itskhoki (2016), (all using different data sources) find that this functional form provides a good approximation of the actual tax and transfer scheme in the US data. In particular, Heathcote, Storesletten, and Violante (2014) and Antrás, De Gortari, and Itskhoki (2016) find very similar estimates of the $\tau_p$ parameter.

For our purpose, there are several weaknesses with this tax and transfer scheme. First, it abstracts from direct forms of compensation that depend upon the circumstances for income loss, such as trade adjustment and assistance programs. For example, the reduction in net taxes for households that experience an income reduction does not depend on weather or not if the loss in income is trade related. It does not provide direct insurance to trade-imposed losses. This is an abstraction in the sense that there are (i) some forms of direct compensation for trade-induced losses through TAAP and (ii) it does not allow us to explore the gains from more direct forms of compensation.\footnote{The TAAP program appears to be quantitatively ineffective relative to traditional social insurance such as social security and disability insurance (see, e.g. Autor, Dorn, and Hanson (2013))}

The final issue is that we are ex-ante restricting the class of instruments the social planner has access to. Thus our analysis leaves open questions if alternative tax instruments may perform better then a non-linear labor income tax. This is an important omission relative to the classic work of Dixit and Norman (1986) who demonstrate how commodity taxation can redistribute the gains from trade in a Pareto improving way.

**Tariffs.** The government imposes a tariff on imported intermediates, $\tau_f$. Mechanically, this value inflates the effective trade costs discussed above. Specifically, the total cost for importing a good will be:

$$\hat{\tau} = \tau(1 + \tau_f).$$

This means that to purchase one unit of the good, $\hat{\tau} > 1$ of the good must be shipped and $(1+\tau_f)$ of the good must be delivered to the “dock.” Out of the units delivered to the dock, $\tau_f$ units are paid to the government and one unit is delivered to the consumer.

While tariffs (in the US context) are generally small, entertaining this policy instrument allows
us to contrast the optimal labor income tax scheme versus a more “isolationist” approach of restricting trade to deal with the ill effects of globalization.

2.3. Households

Within a country, there is a continuum of infinitesimally small households of mass $L$. Each household is infinitely lived and maximizes expected discounted utility

$$E \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t) - B \frac{h_t^{1-\gamma}}{1-\gamma} \right\},$$

where $E$ is the expectation operator and $\beta$ is the subjective discount factor. Period utility depends on both consumption of the final good and the disutility of labor. As we discuss below, we model labor supply as being only on the extensive margin; thus, the parameter $\gamma$ is irrelevant.

Households live and work along the same dimension as the intermediate goods. That is, a household’s location is given by $\omega$—the intermediate goods sector in which it can work.

Given their current location, households can choose to work, to move and work someplace else in the future, and to accumulate a non-state contingent asset. Below, we describe each of these choices in detail.

Working is a discrete choice between zero hours and $\bar{h}$. Thus, the labor supply is purely on the extensive margin. If a household works, it receives income from employment in the intermediate-goods sector in which the household resides. If a household does not work, it receives zero income. In the following presentation, we normalize the value $\bar{h}$ equal to one.

Households can move to an alternative intermediate-goods sector $\omega'$ at some cost. Paying $m$ in units of the final good allows the household to change where it can work in next period. The value of the new location can take several forms. One is the best labor market, as in Lucas and Prescott (1974); an alternative is a random labor market. We focus on the latter specification.

Households residing in a intermediate-goods location face labor income risk associated with fluctuations in local productivity and fluctuations in world prices. We do not allow for any insurance markets against this risk, but let households accumulate a non-state contingent asset $a$ that pays gross return $R$. We treat $R$ as exogenous and not solved for in equilibrium. An interpretation is that this country faces a large supply of assets at this rate. Households face a lower bound on asset holding $-\bar{a}$, so agents can acquire debt up to the value $\bar{a}$.

**State Variables.** The individual state variables of a household are its location $\omega$ and asset holdings $a$. The island-level state variable is the domestic productivity state and world price state. The aggregate state is a distribution over island-level state variables and asset holdings.
Let us expand on this a bit more. The wage per efficiency unit that a household receives is an important island-level object impacting individual decisions. The wage per efficiency unit depends on the value of the marginal product of labor on that island. The marginal product depends on the island’s productivity level. The “value” part depends on (i) the world price for the good produced on the island and (ii) the labor supply decisions of households residing on the island. Given our preference specification in (7), households’ labor supply decisions depend on the distribution of asset holdings within the island. Thus, this is where the aggregate state matters for island-level outcomes.

We focus on a stationary equilibrium. That is, the aggregate state—the distribution over island level states and assets holdings—is constant. Thus, to conserve on notation, we only carry around the households specific state variables: its own asset holdings and island-level state variables associated with its location. In particular, let $s$ denote the domestic productivity and world price combination associated with that island. Furthermore, because the CES aggregator is symmetric over varieties, it is sufficient to index islands by their productivity and world price state. The wage per efficiency unit a household earns is $w(s)$.

**Budget Constraints.** Given the description of the environment, the post-tax earnings of a working household are

$$\tilde{w}(s) = \delta (w(s)\tilde{h})^{1-\tau_p}.$$  

(8)

The household’s period $t$ budget constraint (all denominated in units of the final good) is

$$a_{t+1} + c_t + \iota_{m,t}m \leq Ra_t + \iota_{n,t}\tilde{w}_t(s),$$  

(9)

where the left-hand side are expenditures on new assets, consumption, and possibly moving costs with $\iota_{m,t}$ being an indicator function equaling one if a household moves and zero otherwise. The right-hand side are income payments from asset returns, post-tax income, with $\iota_{n,t}$ being an indicator function equaling one if a household works.

**Recursive Formulation.** The recursive formulation of the household’s problem is

$$V(a, s) = \max [V^{s,w}, V^{s,nw}, V^{m,w}, V^{m,nw}].$$  

(10)

that is a discrete choice among four options: the value of staying and working; the value of staying and not working; the value of moving and working; the value of moving and not working. Unpacking each of these four options is the following. The value of staying and working

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7Given an island with state $s$, denote the measure of agents with asset holdings $a$ as $\lambda(s, a)$. Stationarity implies that this value is constant.
is

$$V^{s,w}(a, s) = \max_{a^\prime \geq -\bar{a}} \left[ u(Ra + \tilde{w}(s) - a^\prime) - B + \beta EV(a^\prime, s^\prime) \right], \quad (11)$$

where $u$ is the utility value over consumption. The value of staying and not working is

$$V^{s,nw}(a, s) = \max_{a^\prime \geq -\bar{a}} \left[ u(Ra - a^\prime) + \beta EV(a^\prime, s^\prime) \right]. \quad (12)$$

The value of moving and working is

$$V^{m,w}(a, s) = \max_{a^\prime \geq -\bar{a}} \left[ u(Ra + \tilde{w}(s) - a^\prime - m) - B + \beta V^m(a^\prime) \right], \quad (13)$$

where there are two key distinctions relative to (11). First, the moving cost, $m$ is paid. Second, the continuation value is $V^m(a^\prime)$ or the value associated with a move. Finally, the value of moving and not working is

$$V^{m,nw}(a, s) = \max_{a^\prime \geq -\bar{a}} \left[ u(Ra - a^\prime - m) + \beta V^m(a^\prime) \right]. \quad (14)$$

### 3. Equilibrium

We close the model by focusing on a small open economy equilibrium. The small open economy assumption is that there is no feedback from home country actions into world prices and the interest rate $R$.

**World Prices.** World prices for commodity $\omega$ evolve according to an AR(1) process in logs:

$$\log p(w)_t = \phi \log p(w)_{t-1} + \epsilon(\omega)_t,$$

where $\epsilon(\omega)_t$ is distributed normally with mean zero and standard deviation $\sigma_w$ and is independent of the innovation to the home country’s productivity $\epsilon_t$.

**A Note on Notation.** We denote $\pi(s)$ as the stationary distribution of productivity states and world prices induced by (2) and (15). And denote $\mu(s)$ as the measure of households working on an island with state $s$.

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8Relative to the trade and labor market dynamics literature, this is similar to the second specification solved in Artuç, Chaudhuri, and McLaren (2010). Moreover, it has the advantage (say, relative to Caliendo, Dvorkin, and Parro (2015)) of being relatively simple, yet allows us to specify about the interaction between trade flows and capital flows.
3.1. Production Side of the Economy

Below, we describe the equilibrium conditions associated with the production side of the economy. These take as given the choices of the household.

**Final Goods Production.** The final-goods producer’s problem is:

$$
\max_{q(s)} P_h Q - \int_s p(s)q(s)\pi(s)ds,
$$

which gives rise to the following the demand curve for an individual variety:

$$
q(s) = \left( \frac{p(s)}{P_h} \right)^{-\theta} Q.
$$

where $Q$ is the aggregate demand for the final good; $P_h$ is the price associated with the final good which will be carried around briefly, but is ultimately normalized to the value one.

**Intermediate Goods Production.** The intermediate-goods-producer’s problem is

$$
\max_{q(s),\ell(s)} p(s)q(s) - w(s)\ell(s)
$$

or to choose the quantity produced to maximize profits. Competition implies that the wage per efficiency unit (in units of the final good) at which a firm hires labor is:

$$
w(s) = p(s)z
$$

or the value of the marginal product of labor. Only at the wage in (19) are intermediate-goods producers willing to produce.

**Intermediate Goods, International Trade, and Market Clearing.** To formulate the pattern of trade, we denote the set of prevailing prices that the final-goods producer in the home country faces as $p(s)$, $\hat{\tau}p_w$. The final-goods producer purchases intermediate goods from the low-cost supplier. This decision gives rise to three cases with three different market-clearing conditions: if the good is non-traded; if the good is imported; and if the good is exported.  

Below, we describe demand and production in each of these cases.

- **Non-traded.** If the good is non-traded, then the domestic price for the home country must satisfy the following inequality: $\frac{p_w}{\tau} < p(s) < \hat{\tau}p_w$. That is, from the home country’s perspective, it is optimal to source the good domestically and not optimal for the home

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9This is more nuanced than the standard formulation in Eaton and Kortum (2002) due to the frictional labor market. In our model, there are situations in which an intermediate good is both imported and produced domestically, which is not the case in the Eaton and Kortum (2002) model.
country to export the product.

In this case, the market-clearing condition is:

\[
\left( \frac{p(s)}{P_h} \right)^{-\theta} Q = z \left( \mu(s) / \pi(s) \right)
\]  \hspace{1cm} (20)

or that domestic demand equals production. The left-hand part is demand and the right-hand side is supply. That is the productivity of domestic suppliers multiplied by the supply of labor units in that market.

**Imported.** If the good is imported, then the domestic price for the home country must be \( p(s) = \hat{\tau} p_w \). Why? If the price were lower, then it would not be imported. If the domestic price were higher, then the good will be imported with not domestic production and, thus, the prevailing domestic price will equal the imported price. With frictional labor markets, there may be some domestic production so the quantity of imports is

\[
\left( \frac{\hat{\tau} p_w}{P_h} \right)^{-\theta} Q - z \left( \mu(s) / \pi(s) \right) > 0.
\]  \hspace{1cm} (21)

That is home demand (net of home production) is met by imports of the commodity, net of tariffs. Rearranging gives

\[
\left( \frac{\hat{\tau} p_w}{P_h} \right)^{-\theta} Q = z \left( \mu(s) / \pi(s) \right) + (1 - \tau_f) \text{imports}(s)
\]  \hspace{1cm} (22)

or domestic demand equals domestic production plus imports net of tariffs.

**Exported.** If the good is exported, then the prevailing price must be \( p(s) \tau = p_w \). Why? If the home price were larger, then the good would not be purchased on the world market. And the price can not be lower, as arbitrage implies that the price of the exported good sold in the world market must equal the prevailing price in that market. Finally, note that only the trade cost, not the tariff, matters here. At this price, the quantity of exports is

\[
\left( \frac{p_w / \tau}{P_h} \right)^{-\theta} Q - z \left( \mu(s) / \pi(s) \right) < 0
\]  \hspace{1cm} (23)

or domestic demand net of production which should be negative, implying that the coun-
try is an exporter. Rearranging gives
\[
\left( \frac{p_w}{P_h} \right)^{-\theta} Q = z (\mu(s)/\pi(s)) - \text{exports}(s) \tag{24}
\]
or domestic demand equals domestic production minus exports.

**The Final Good and Market Clearing.** The final good’s producer sells the final good to consumers. Thus, we have the following market-clearing condition
\[
Q = C + G = \int_s \int_a c(s, a) \lambda(s, a) da ds + G, \tag{25}
\]
where \(c(s, a)\) is the consumption policy function that satisfies the households’ problem, and \(\lambda(s, a)\) is the mass of consumers with state \(s\) and asset holding \(a\) (defined below in (26)). This relationship says that household-level consumption—aggregated across all households—plus government consumption must equal the aggregate production of the final good \(Q\).

Market-clearing conditions for the intermediate goods in (20),(22), (24) and the aggregate final good in (25) summarize the equilibrium relationship on the production side of the economy.

### 3.2. Household Side of the Economy

The households in the economy make choices about where to reside, how much to work, and how much to consume. Here, we describe the equilibrium conditions associated with these choices. In the discussion below, we define the following functions—\(\{\iota_m(s, a), \iota_n(s, a), g_a(s, a)\}\)—as the move, work, and asset policy functions that satisfy the households’ problem in (10).

**Population and Labor Supply.** We define the probability distribution of households across assets and states as \(\lambda(s, a)\). Furthermore, define the probability distribution of households in the next period as \(\lambda'(s, a)\). The distribution of households evolves across time according to the following law of motion:
\[
\lambda'(s', a') = \int_s \int_a \lambda(s, a)(1 - \iota_m(s, a))\pi(s', s) + \lambda(s, a)\iota_m(s, a)\bar{\pi}(s') da ds. \tag{26}
\]
Equation (26) says the following: in the next period, the mass of households with asset holdings \(a'\) in state \(s'\) equals the mass of household that do not move multiplied by the transition probability that \(s\) transits to \(s'\). This is the first term in equation (26). Plus the mass of households that do move, multiplied by the probability that they end up in state \(s'\). This is the second term in equation (26). The probability, \(\bar{\pi}(s')\), is given by the moving protocol—i.e., random assignment across islands according to the invariant distribution associated with \(\pi(s', s)\). All of this is
conditional on those households that choose asset holdings equal to \( a' \). This is denoted by the conditionality under the integral sign.

Given a distribution of households, the supply of labor to intermediate good producers with productivity state \( s \) is,
\[
\int_a \ell_n(s, a)\lambda(s, a) da = \mu(s),
\]
which is the size of the population residing in that market multiplied by the labor supply policy function and integrated over all asset states. This, then, connects the supply of labor with production in (20)-(24).

**Asset Holdings and Consumption.** The distribution of asset holdings and consumption take the following form. Next period, aggregate net-asset holdings are
\[
\mathcal{A}' = \int_a \int_s g_a(s, a)\lambda(s, a) ds da.
\]
A couple of points about this are warranted. First, this is in aggregate—some households in the home country may have positive holdings, while others may have negative holdings. Second, net asset holdings must always be claims on foreign assets since there is no domestic asset in positive supply (such as capital).

Using the definition in (28) we can work from the consumers’ budget constraint and derive aggregate consumption:
\[
C = -\mathcal{A}' + RA + \int_a \int_s \left\{ \tilde{w}(s)\ell_n(s, a) - m\ell_m(s, a) \right\} \lambda(s, a) ds da.
\]
In words, aggregate consumption equals net asset purchases (the first two terms) plus wage income net of moving costs.

### 3.3. Government

We assume that the government runs a balanced budget. Thus,
\[
G = \int_a \int_s T(w(s))\ell_n(s, a)\lambda(s, a) dsd da + \tau_f \int_s p(s)\text{imports}(s) ds,
\]
which says the following: Government spending must equal: (i) labor income tax revenues conditional on working and then integrating over all markets and asset states; plus (ii) tariff revenue from imports.

What does the government do in our economy? The spending level \( G \), the tax progressivity \( \tau_p \),
parameter, and the tariff rate are exogenously given. The government then picks the average tax rate, \( \delta \), such that (30) holds.

### 3.4. A Stationary Small Open Economy (SSOE) Equilibrium

Given the equilibrium conditions from the production and household side of the economy, we define a “Stationary Small Open Economy (SSOE) Equilibrium” equilibrium.

**A Stationary Small Open Economy (SSOE) Equilibrium.** Given world prices \( \{ p^w, R \} \) and government policy \( \{ G, \tau_p, \tau_f \} \), a stationary Small Open Economy Equilibrium is domestic prices \( \{ p(s) \} \), tax rate \( \delta \), policy functions \( \{ g_a(s, a), \iota_n(s, a), \iota_m(s, a) \} \), and a probability distribution \( \lambda(s, a) \) such that

1. Firms maximize profits, (16) and (18);
2. The policy functions solve the household’s optimization problem in (10);
3. Demand for the final and intermediate goods equals production, (20), (21), (23) and (25);
4. The government budget is balanced (30);
5. The probability distribution \( \lambda(s, a) \) is a stationary distribution associated with
   \( \{ g_a(s, a), \iota_m(s, a), \pi(s', s) \} \). That is, it satisfies
   \[
   \lambda(s', a') = \int \int \lambda(s, a)(1 - \iota_m(s, a))\pi(s', s) + \lambda(s, a)\iota_m(s, a)\pi(s') da ds.
   \]

The idea behind the equilibrium definition is the following. The first bullet point (i) gives rise to the equilibrium conditions for the demand of intermediate goods in (17) and wages (19) at which firms are willing to produce. The second bullet point (ii) says that households are optimizing.

At a superficial level, bullet (iii) says that demand must equal supply. It’s meaning, however, deeper. The households’ choices of the matter for both the demand and the supply side. Specifically, it requires that prices (and, hence, wages) must induce a pattern of (i) consumption and (ii) labor supply such that demand for goods equals the production of goods.

Bullet point (v) requires stationarity. Specifically, the distribution of households across productivity and asset states is not changing. Mathematically, this means that distribution \( \lambda(s, a) \) must be such that when plugged into the law of motion in (26), the same distribution is returned.

Finally, note that there is no requirement that the asset market clears—i.e., that (28) equals zero. This is an aspect of the small open economy assumption. At the given world interest rate
$R$, the assets need not be in zero net supply. This implies that trade need not balance, as the trade imbalance will reflect asset income on foreign assets and the acquisition of assets. After adjusting for moving costs, this implies that the current account and capital account are always zero in a stationary equilibrium, but that trade may be imbalanced.

**Computation.** Computing a stationary equilibrium for this economy deserves some discussion. First, this economy is unlike standard incomplete markets models in which only one or two prices (e.g., one wage per efficiency unit and/or the real interest rate) must be solved for. In contrast, we must solve for an equilibrium function $p(s)$. Thus, the iterative procedure is to (i) guess a price function; (ii) solve the household’s dynamic optimization problem; (iii) construct the stationary distribution $\lambda(s, a)$; (iv) check whether markets clear; and (v) update the price function. See, e.g., Krusell, Mukoyama, and Şahin (2010), who solve a similar problem.

Second, an important observation is that the inequalities in (21) and (23) impose additional structure on an equilibrium. The key observation is that when domestic demand and supply are not equal, the price in those markets must respect bounds on international arbitrage. This implies that the problem of finding a price function consistent with a stationary equilibrium can be represented as a mixed complementarity problem (see, e.g., Miranda and Fackler (2004)). Appendix B provides a complete description of our solution procedure.

4. Model Properties

This section describes some qualitative properties of the model. It borrows from our own parallel work in Lyon and Waugh (2018), which studies the workings of the model. Below we focus on two issues: (i) the pattern of trade across labor markets; and (ii) how trade exposure affects wages and, in turn, the desire for social insurance.

4.1. Trade

To illustrate the pattern of trade across islands, first define the following statistic:

$$\omega(s) := \frac{p(s)z\mu(s)}{p(s)z\mu(s) + p(s)\text{imports}(s) - p(s)\text{exports}(s)}.$$ (32)

What does equation (32) represent? The denominator is the value of domestic consumption: everything domestically produced plus imports minus exports. The numerator is production. The interpretation of (32) is how much of domestic consumption at the island/variety level the home country is producing. This is similar to the micro-level “home share” summary statistic emphasized in Arkolakis, Costinot, and Rodríguez-Clare (2012). As we discuss below, this statistic (i) provides a clean interpretation of a labor market’s exposure to trade and (ii) is tightly connected with local labor market wages.
Figure 1 plots the home share (raised to the power of inverse \( \theta \)) by world price and home productivity. There are three regions to take note of: where goods are imported, exported, and non-traded. First, in the regions where the home share lies below one, demand is greater than supply, and, hence, goods are being imported. This region naturally corresponds to the situation with low world prices or low home productivity—i.e. the economy has a comparative disadvantage in producing these commodities.

Second, in the regions where the home share lies above one, supply is greater than demand, and, hence, goods are being exported. This region corresponds to high world prices or high home productivity. In other words, this is where the country has a comparative advantage and is an exporter of the commodities.

Third, there is the “table top” region in the middle, where the home share equals one. Hence, this is the region where the goods are non-traded. Exactly like the inner, non-traded region in the Ricardian model of Dornbusch, Fischer, and Samuelson (1977), the reason is trade costs. In this region, world prices and domestic productivity are not high enough for a producer to be an exporter of these commodities given trade costs. Furthermore, world prices and domestic productivity are not low enough to merit importing these commodities either. Thus, these goods are non-traded.

Finally, unlike Dornbusch, Fischer, and Samuelson (1977) or Eaton and Kortum (2002), it is important to reflect on the stochastic nature of this economy. While the stationary equilibrium of the economy leads to the stationary pattern of trade seen in Figure 1, individual islands transit between different states (world prices and domestic productivity). For example, an island may be an exporter, but given a sequence of bad productivity shocks, the island will stop exporting and maybe even become an importer of a commodity it once exported.

4.2. Trade and Wages

One can connect the pattern of trade across labor markets in Figure (1) with the structure of wages in the economy. As we show in the Appendix and in Lyon and Waugh (2018), pre-tax real wages in a market with state variable \( s \) equal

\[
w(s) = \omega(s) \frac{1}{\theta} \frac{\mu(s)}{\pi(s)}^\frac{1}{\theta} \pi \frac{1}{\theta} C^\frac{1}{\theta}. \tag{33}\]

Here \( \omega(s) \) is the home share defined in (32); \( \mu(s) = \frac{\mu(s)}{\pi(s)} \) is the number of labor units; \( z \) is domestic productivity; \( C \) is aggregate consumption.

Equation (33) connects the trade exposure measure in (32) with island-level wages. A smaller home share implies that wages are lower with elasticity \( \frac{1}{\theta} \). This means that if imports (relative to domestic production) are larger, then wages in that labor market are lower. Similarly, a larger
Figure 1: Trade: Home Share, $\omega(s)^{\frac{1}{\pi}}$

Figure 2: Wages
home share means that wages are higher.

While this looks like the “micro-level” analog of the aggregate result of Arkolakis, Costinot, and Rodríguez-Clare (2012) it is different in one important respect: the micro-level wage response to micro-level trade exposure takes the opposite sign—a home share leads to lower wages.\(^{10}\) A way to understand this result is as follows: Wages reflect the value of the marginal product of labor. In import competing islands, trade results in lower prices and, hence, the “value” part of the value of the marginal product of labor falls resulting in lower wages. The CES demand system provides a tight link between prices and shares, hence the way that the home share enters into (33).

This result has a close relationship to the Specific Factors Model of trade (Samuelson (1971), Jones (1971)). Since labor is fixed (within the period), it is as if labor is the “specific factor” to the island (rather than capital or land in the textbook treatment). Like in the textbook model, trade lowers the relative price of the imported good, lowering the relative value of the marginal product of that specific factor.

Figure 2 illustrates these observations by plotting the logarithm of pre-tax wages by world price and home productivity so it exactly matches up with Figure 1. As equation (33) makes clear, there is a tight correspondence between wages and the home share in Figure 1. As in Figure 1, there are three regions to take note of. The first region is where import competition is prevalent (low world prices or low home productivity) wages are low. The second region is where exporting is prevalent. Exporting regions are able to capture high world prices, and, thus, wages are high in these islands. Finally, the center region is where commodities are non-traded. Here, the gradient of wages very much mimics the increase in domestic productivity. In contrast, where goods are imported or exported, the wage gradient mimics the change in world prices.

Equation (33) connects with the aggregate gains from trade. Any change in aggregate trade exposure will also change in aggregate consumption, i.e. the \(C\) term. That is all workers benefit from the “aggregate gains to trade”, but the island-level incidence varies with its trade exposure and may mitigate or completely offset the aggregate benefits from trade.

Again, it is important to reflect on the stochastic nature of this economy. While the stationary equilibrium of the economy results in a stationary distribution of wages, individual islands (and households living on those islands) transit between different states (world prices and domestic productivity). For example, an island may be an exporter with households receiving high wages, but given a sequence of bad productivity shocks, the island will stop exporting.

\(^{10}\)The sign also takes on the opposite value relative to new quantitative geography models (see, e.g., Redding (2016)). In these models, each location can produce all goods, so each location behaves in a similar manner to Eaton and Kortum (2002).
and household wages will fall.

Outcomes of this nature—a sequence of uninsurable, bad shocks—motivate the desire for social insurance. International trade changes the labor market consequences of various shocks, which, in turn, change the demand for social insurance. Next we quantitatively weigh the benefits of social insurance via progressive labor income taxation with the associated costs that come from distorting labor market participation decisions.

5. Calibration

Our calibration approach is rather pedestrian. We use existing parameter values from various literatures and then picks several parameter values so that the model replicates certain aggregate and cross-sectional moments in the US data. While we select parameters to target all moments simultaneously, we choose each target moment with a specific parameter in mind as outlined below.

**Time Period and Geography.** The time period is set to a year. Geographically, in our model, there is an abstract notion of an island, households living on that island, and working within its labor market. For the purposes of disciplining the pattern of migration, we will think about the empirical counterpart to an island as a Commuting Zone (see Tolbert and Sizer (1996)) and as used in Autor, Dorn, and Hanson (2013)).

Alternatives would be to think of an island as some cross between physical geography and an occupation/industry. The benefit of our choice is that it provides a direct connection to Autor, Dorn, and Hanson’s (2013) evidence on trade’s effects at the commuting-zone level. Given the above discussion about equation (33), this is consistent with their finding that commuting zones with increases in import exposure experience relative wage losses. The cost of this choice is that alternative perspectives on geography in our model may affect how much income smoothing households can achieve through changing, say, occupations. And, in turn, this would affect the desirability of social insurance.

**Preferences.** Given the time period we set discount factor equal to 0.95. Given our specification in (7) and the restriction on labor supply, there is only one parameter to calibrate: \( B \), which controls the disutility of working. We pick the disutility term \( B \) to match a labor force participation rate of 67 percent which corresponds with the average value across the period of 1990-2000 in US data.

Related to this discussion is a question of the elasticity of labor supply. As in Rogerson and Wallenius (2009) and Chang and Kim (2007), our restrictions on the choice sets of households implies that at the micro-level, intensive margin labor supply elasticity is set to zero. In other words, conditional on participation, changes in labor earnings will not affect, say, hours worked.
While extreme, this specification is consistent with a wide range of studies that typically find low elasticities (see, e.g., the evidence discussed in Keane (2011)). With that said, the micro-labor supply elasticity can diverge from the aggregate labor supply elasticity due to movements in and out of the labor force, i.e., the extensive margin.

**Financial Constraints.** The borrowing limit parameter is calibrated to match properties of the aggregate wealth distribution. Krueger, Mitman, and Perri (2016) report from the Survey of Consumer Finances that approximately 40 percent of households have zero or negative wealth. Thus, we choose the borrowing limit so that the model replicates this fact.

**Productivity and World Prices.** The productivity process in (2) and (15) leave three parameters to be calibrated: \( \{ \phi, \sigma_z, \sigma_w \} \), the parameter controlling the persistence of the shocks and the size of the innovations.

We simplify the process more and restrict the standard deviation of innovations to productivity to be the same size as the standard deviation of innovations to world prices. A specification of this nature would make sense in a symmetric two-country world.

Given this restriction, we use existing estimates of labor income processes to discipline these parameters. Specifically, we use the estimates from Kaplan (2012), which imply a value of 0.95 for \( \phi \) and a value of 0.026 for \( \sigma_z \). Given the relationship between wages and productivity described above, much of the economy will not be import- or export-exposed; thus, wages will mimic the productivity process \( z \) adjusted for the value of \( \theta \). Therefore, it is natural to have our model to replicate these features of the data.

With that said, a key issue in this class of models is how persistent the shocks are and, more specifically for our question, the permanence of the change in comparative advantage. This is important in that it will affect how insurable or uninsurable these shocks are and, thus, the desirability of progressive taxation and social insurance. We speculate that the results of Krishna and Senses (2014) and Hanson, Lind, and Muendler (2015) speak to these dynamics of comparative advantage, as well.

These parameters also play a deeper and less obvious role in determining how elastic aggregate trade flows are to a change in trade frictions. Much like in the model of Eaton and Kortum (2002), the extent of technology heterogeneity controls how elastic trade flows are to changes in trade costs. Furthermore, per the insights of Arkolakis, Costinot, and Rodríguez-Clare (2012), these parameters also partially control the aggregate gains from trade.

The final world price that we must calibrate is the gross real interest rate, \( R \). We set this equal to 1.02, which corresponds with a two percent annual interest rate.

**Migration Cost and Location Choice:** We choose the migration cost to match the aggregate data on migration rates across commuting zones. We use the the IRS migration data which
Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target Moment/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor, $\beta$</td>
<td>0.95</td>
<td>—</td>
</tr>
<tr>
<td>Persistence of $z$ and $p_w$ process</td>
<td>0.95</td>
<td>—</td>
</tr>
<tr>
<td>Std. Dev. of innovations to $z$ and $p_w$</td>
<td>0.17</td>
<td>—</td>
</tr>
<tr>
<td>World Interest Rate, $R$</td>
<td>1.02</td>
<td>—</td>
</tr>
<tr>
<td>Tax Progressivity</td>
<td>0.18</td>
<td>Heathcote, Storesletten, and Violante (2014)</td>
</tr>
<tr>
<td>Demand Elasticity</td>
<td>4.00</td>
<td>—</td>
</tr>
<tr>
<td>Disutility of work, $B$</td>
<td>1.51</td>
<td>Aggregate participation rate, 66 %</td>
</tr>
<tr>
<td>Migration Cost, $m$</td>
<td>0.85</td>
<td>Cross CMZ. migration rate, 3%</td>
</tr>
<tr>
<td>Borrowing Limit, $-\bar{a}$</td>
<td>0.45</td>
<td>40 % households with $\leq 0$ net worth</td>
</tr>
<tr>
<td>Trade Cost</td>
<td>2.32</td>
<td>Imports to GDP ratio 10%</td>
</tr>
</tbody>
</table>

uses the address and reported income on individual tax filings to track how many individuals move in or out of a county. We compute that a bit over three percent of households move across a commuting zone at a yearly frequency. We pick the migration cost to target this value. This is slightly larger than the values reported in Molloy, Smith, and Wozniak (2011).

The eighth row of Table 1 reports the size of the moving cost. In the baseline economy, the size of the moving cost is about 60 percent of average (yearly) income. This is large, but substantially smaller than the costs estimated in Kennan and Walker (2011) who find they are on the order of about three hundred thousand dollars.

A related issue is the specification of where moving households end up. As discussed above, we use the random labor market specification. That is upon moving, a household will end up in a random labor market, with the distribution function being the invariant distribution of labor markets. This is a simplification, but a more flexible specification would allow us to think about the worker-level evidence of Autor, Dorn, Hanson, and Song (2014) and the repeated exposure of certain workers to trade shocks.

**Tax Function and Government Spending.** We use the estimates from Heathcote, Storesletten, and Violante (2014) and set $\tau_p$ equal to 0.18. Using an alternative data set from the Congressional Budget Office, Antràs, De Gortari, and Itskhoki (2016) find a very strong fit of the tax function in (4) and a similar estimate of progressivity. Guner, Kaygusuz, and Ventura (2014) provide an exploration of these issues as well and find similar performance of fit and estimates.
of progressivity.

We set government spending, $G$, to be 19 percent of GDP. This is consistent with National Accounts data for the US over the past 20 years. Recall that the parameter $\delta$ on the tax function is determined in equilibrium so that the government runs a balanced budget.

**Demand Elasticity $\theta$.** Consistent with a wide range of estimates, we set its value equal to four. The trade literature has put much effort into estimating this parameter, and the value four lies within the middle of the range of recent estimates. At the lower end of the range are the estimates from Broda and Weinstein (2006) who find that the median elasticity across product categories is around three. At the upper end of the range are aggregate estimates from Parro (2013) and Caliendo and Parro (2014); using aggregate tariff and trade flow data, they find values near five. Simonovska and Waugh (2014a) and Simonovska and Waugh (2014b) find values that lie in the middle around four.

**Trade Costs and Tariffs.** Tariffs are set to value zero. This is a rough approximation of current observed, trade weighted tariffs in the US. We choose the iceberg trade cost to target an aggregate import to GDP ratio of ten percent. This is consistent with the aggregate level of trade in in the mid-1990’s, pre-China WTO accession.

### 6. Optimal Progressivity and Openness to Trade

Given the calibrated model, we ask several questions of it. First, what trade-offs does the policy maker face? Second, how does the optimal tax policy change with increased openness to trade? Third, how does openness change the benefits of a progressive tax system? Below, we describe the social welfare function that the policy maker is maximizing and then answer each of these questions.

#### 6.1. Social Welfare Function

We focus on a utilitarian planner that places equal weight on households within the domestic economy. That is,

$$W(\tau_p, \tau) = \int_{s} \int_{a} V(a, s) \lambda(s, a) da ds,$$

or the value function of a household integrated over markets and asset states with respect to the stationary distribution across those states $\lambda(s, a)$. No weight is given to foreign agents. Here, we explicitly index social welfare by the tax progressivity parameter and by the trade cost. Thus, given the social welfare function in (34), the optimal degree of progressivity is

$$\tau_p^*(\tau) = \arg \max W(\tau_p, \tau)$$

(35)
that is, the tax progressivity parameter that maximizes social welfare. Here, we make explicit that the tax progressivity parameter depends on the trade cost or the extent to which the economy is open or closed. When presenting results, we convert welfare units into consumption-equivalent values—that is, the permanent percent change in consumption that must be allocated to make a household indifferent between living in the baseline economy and in an economy with an alternative progressivity parameter $\tau_p$.

When we compare social welfare across different levels of openness, we do not consider the transition dynamics associated with the move to a more open regime. In Lyon and Waugh (2018), we find that welfare gains and costs of a trade shock are substantially larger and more dispersed once transition dynamics are taken into account. In particular, for certain segments of the population, the initial adjustment is quite costly. Given discounting, this suggests that consideration of transition dynamics would strengthen the argument for a more progressive tax system.

6.2. Optimal Progressivity and the Insurance, Misallocation Trade-off

What trade-offs does the planner face? Generally, these issues are well understood—the planner trades off the gains from social insurance versus the costs of distorting incentives. Unique to our environment is that the key margin being distorted is the migration of labor across space which leads to spatial misallocation and losses in allocative efficiency.

We explore these questions by tracing out social welfare for different levels of $\tau_p$ or tax progressivity. All other parameters are fixed. Furthermore, we report welfare relative to the baseline economy, thus, when $\tau_p = 0.18$, the welfare gain equals zero.

Figure 3 shows that social welfare displays an inverted “U” shape as progressivity varies. Optimal progressivity is the progressivity parameter associated with the peak of the inverted U.\textsuperscript{11} The reason behind the inverted U shape in Figure 3 is the trade-off between providing better social insurance and distorting labor supply and migration and, hence, reducing the size of the “pie”. Figure 3 illustrates this point by plotting output versus tax progressivity. As progressivity increases, output systematically declines.

To understand the mechanics as to why output is falling, we can perform the following decomposition of GDP in our model. Aggregate GDP in the economy is:

$$Y = \int w(s)\mu(s)\pi(s)ds,$$

\textsuperscript{11}In our calibration, the US economy lies to the left of the optimal policy. In particular, the optimal progressivity is found to be 0.27, versus the current progressivity of 0.18 as measured by Heathcote, Storesletten, and Violante (2014). However, the welfare gains from a move to an optimally progressive system are very small—less than one tenth of a percent.

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Figure 3: Social Welfare and Progressivity

Figure 4: GDP, GDP Decomposition, and Progressivity
that is the sum over labor earnings multiplied by the number of workers earning those earnings, i.e. total payments to labor. Following, Olley and Pakes (1996), we can express the left-hand side of (36) as

\[ Y = \bar{w} \bar{\mu}(s) + \int_s (w(s) - \bar{w}) (\mu(s) - \bar{\mu}(s)) \pi(s) ds, \]

(37)

where \( \bar{w} \) is the average wage; \( \bar{\mu}(s) \) is aggregate labor supply. The first term in (37) would arise in a frictionless framework—wages are equalized across locations and hence aggregate output equals the wage multiplied by aggregate labor supply. The second term in (37) reflects allocative efficiency. It is only present if there is some form of misallocation, i.e., wages are not equalized across islands.\(^{12}\) Moreover, since it is a covariance, it quantifies the extent to which places with higher labor earnings have more people working. In other words, this term answers the question: Are households allocated more or less efficiently?

Figure 4 plots output (blue solid) and the covariance term in (37) (red dashed). To the right of current policy, that output systematically declines by large amounts. For example, a move from baseline policy to progressivity parameter of 0.40 leads to a two percent decrease in output relative to the baseline. A decrease in allocative efficiency accounts for almost all of the decrease in output.

Why is allocative efficiency falling? Because increased provision of social insurance reduces migration. Figure 5 plots the percentage change in migration rates relative to the baseline economy. Here, there are very large declines in migration as the tax system becomes more progressive. Migration is important for allocative efficiency. When migration rates are high, more households move out of low productivity, low-comparative advantage islands into islands with more favorable labor market conditions. As migration rates decline, allocative efficiency starts to fall and, in turn, output and aggregate productivity decline.

The reason for the fall in migration is because the need to migrate falls with the increased provision of social insurance. Households motive for migration is to avoid adverse shocks to find a better income realization (see, e.g., Lagakos, Mobarak, and Waugh (2017)).\(^{13}\) As the tax system becomes more progressive, this provides households with better insurance and, thus, households migrate less often.

\(^{12}\)The fact that this last term is active, suggests that our framework (this paper and Lyon and Waugh (2018)) has an additional gains from trade relative to, say Eaton and Kortum (2002). If opening to trade incentives households to reallocate, then opening to trade will leads to a gains through a terms of trade effect and a new allocative efficiency term which is not present in efficient frameworks (e.g., Eaton and Kortum (2002)).

\(^{13}\)This mechanism is in contrast to the motive for moves in the stationary equilibrium of Artuç, Chaudhuri, and McLaren (2010) and Caliendo, Dvorkin, and Parro (2015). While similar in spirit to this model, labor earnings across regions/sectors are constant, and households move dynamically because of shocks to preferences, not because of unexpected shocks to labor earnings as in our model.
The loss in output is not about labor supply. To see this, first note that aggregate labor supply effects would only show up in the first term in (37). Given that most of the movement is in the allocative efficiency term, this implies labor supply is not playing an important role. Figure 3 further confirms this point by plotting labor supply. Labor supply changes by less than half a percentage point over the whole span of tax progressivity parameters.

There are several reasons why labor supply is not that responsive. First, the aggregate, average tax rate is not changing across tax regimes because the rate of government spending pins this value down and it is held constant throughout. Thus, any aggregate labor supply must come from only distributional effects. Second, the distributional effects are likely to be small as labor supply at the micro level is inelastic. This largely follows from our choice to model labor supply as being purely on the extensive margin.

6.3. How Does the Optimal Policy Change with Openness to Trade?

To answer this question, we hold all calibrated values fixed, but change trade costs to target several different regimes of openness. That is, we find the trade costs such that the calibrated economy delivers a 20, 30, and 40 percent import to GDP ratio. Then, for different regimes, we (i) trace out social welfare as a function of the tax progressivity parameter; and (ii) measure the
Table 2: Openness and Optimal Progressivity

<table>
<thead>
<tr>
<th>Imports/GDP</th>
<th>$\tau^*_p$</th>
<th>Gains from $\tau^*_p$</th>
<th>Losses from Flat</th>
<th>Welfare Marginal Tax Rate</th>
<th>90th Prct.</th>
<th>10th Prct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.27</td>
<td>0.10</td>
<td>-0.83</td>
<td>48.0</td>
<td>27.3</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>0.32</td>
<td>0.34</td>
<td>-1.39</td>
<td>52.8</td>
<td>25.9</td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>0.37</td>
<td>0.72</td>
<td>-1.94</td>
<td>56.8</td>
<td>25.5</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.45</td>
<td>1.38</td>
<td>-2.62</td>
<td>63.5</td>
<td>24.0</td>
<td></td>
</tr>
</tbody>
</table>

Note: 90th Prct is the 90th percentile of the labor income distribution; 10th is the 10th percentile. Gains are consumption equivalent values between living in the baseline economy and an economy with an alternative progressivity parameter $\tau^*_p$.

Figure 6 plots social welfare as a function of tax progressivity for several different regimes (as before, we plot everything in percent difference from the current policy). The red circle on each curve of Figure 6 plot the optimal values. The second column in Table 2 reports optimal values.

We find that the tax system should become more progressive as the economy becomes more open. For example, Table 2 shows that optimal progressivity steadily increases when moving above recent levels of US openness (an import share of ten percent). Graphically, this is shown by noting how the large red dots in Figure 6 systematically move right and up as the economy becomes more open.

The third column in Table 2 reports the welfare gains from moving to an optimal policy. First, the gains from moving towards the optimal policy are increasing with the level of openness. With that said, only at relatively large levels of openness (at least for the US, but comparable to economies such as Canada and Mexico) do we see quantitatively large welfare gains in moving towards the optimal policy.

These prescribed changes in progressivity map into large changes in tax rates as the economy becomes more open. Figure 7 reports marginal tax rates relative to income percentile in the economy. Marginal tax rates become lower for those at the bottom of the distribution and systematically higher for those at the top of the distribution. The last two columns of Table 2 report the the marginal tax rates of those in different labor income percentiles in the optimal tax system. In an economy with a ten percent Import/GDP ratio, those in the top 10th percentile
Figure 6: Social Welfare and Progressivity for Different Levels of Openness

Figure 7: Optimal Marginal Tax Rates and Income for Different Levels of Openness
of the labor income distribution have an marginal tax rate of 48 percent, the marginal tax rate for those at the bottom of the distribution is about 27 percent. In the more open economy, with an import share of 40 percent, marginal tax rates of those in the top 10th percentile increase to 63 percent, those at the bottom 10th percentile decrease to 24 percent.

Overall, the elasticity of top tax rates to openness is 1/2. That is optimal policy dictates that a ten percentage point increase in imports as a fraction of GDP necessitates a five percentage point increase in marginal tax rates for those in the 90 percentile of the income distribution.

The reason for increasing progressivity with openness is because the costs of progressive taxation are relatively constant across different levels of openness, but the benefits increase with openness. Thus the cost-benefit analysis tilts towards becoming more progressive as the economy opens.

To see this, Figure 8 plots GDP for different levels of openness (relative to the baseline tax system); red dots indicated points corresponding with optimal progressivity. The important observation is that curves essentially all lie on top of each other. That is the output cost of progressivity does not increase as the economy becomes more open. If anything, Figure 8 suggests that the output cost decreases slightly.\footnote{We speculate this is because openness to trade increases allocative efficiency as it incentives more migration. Hence, the planner has more scope to provide social insurance.}

The benefits from progressive taxation are increasing with openness. The mechanism is that increased openness changes the nature of uninsurable income risk, and this motivates the increased provision of social insurance. For example, the variance of log pre-tax labor income systematically rises from 0.13 in the baseline economy to 0.25 in the economy with a import to GDP share of 40 percent. This change in the nature of income risk increases precautionary savings motives (see, e.g., Huggett and Ospina (2001) and, in turn, the desire for social insurance. This mechanism is consistent with the findings of Krishna and Senses (2014).

Finally, the result that optimal progressivity should increase with openness is worth comparing to that of Antràs, De Gortari, and Itskhoki (2016). In contrast to our results, they find that tax progressivity should decrease as the economy opens up. Like their model, our trade setting has a close mapping to modern, quantitative trade frameworks (in our case Eaton and Kortum (2002), and in theirs, Melitz (2003)). We speculate that the key difference is that we focus on a different benefit of redistributive taxation. In our model, the motive for redistributive taxation arises from risk and incomplete insurance. Trade changes the nature of uninsurable income risk, and this motivates the increased provision of social insurance. This insurance motive is absent from Antràs, De Gortari, and Itskhoki (2016). In their setting, trade benefits all agents (see, e.g., Proposition 4) but generates increases in earnings inequality; aversion to inequality is the only reason to engage in redistributive taxation.
6.4. How Does Openness Change the Benefits of a Progressive Tax System?

While the welfare gains from moving toward the optimal policy may be small, this does not imply that a progressive tax policy is not beneficial as the economy opens to trade. In fact, a progressive tax system plays an important role in an open economy. To illustrate this point, we ask what the benefits of a progressive tax system are relative to a flat tax system and how they change with the level of openness.

Figure 6 shows that the social welfare curves become more concave as the economy opens up. Relative to a flat tax system, this means the benefits of a progressive tax system become larger as the economy becomes more open. Unlike the benefits associated with the optimal policy, the gains are large even for current levels of openness. Table 2 reports that a move to a flat tax system at current levels of openness would lower welfare by almost one percent. For higher levels of openness, a move to a flat tax system would erode welfare by more than two and half percent.

This result implies that the existing tax system complements the traditional gains from trade. With a flat tax system, the gains from trade associated with a move from a ten to 20 percent import share lead to about a five and a half percent gain in welfare. However, if the tax system is more progressive, even at the baseline of $\tau_p = 0.18$, the welfare gains are just over seven percent.
In other words, the gains from trade increase by 25 percent with increases in progressivity.

This final point provides an interesting perspective on the historical evolution of tax policy and openness to trade. Antràs, De Gortari, and Itskhoki (2016) find that the US tax system has declined in its progressivity—e.g., the measure $\tau_p$ declined from 0.25 in 1980 to 0.16 in 2005. During this same time period, imports over GDP essentially doubled. That is as the US economy has open to trade, its tax system has become more regressive. Viewed through the lens of our model, this suggest that US tax policy was moving in the wrong direction given trends in globalization. Moreover, the US has not gained what it could have from trade due to changes in tax policy.

6.5. How Does a Progressive Tax System Compare to Tariffs?

How does a progressive tax system compare to import tariffs? This comparison is of interest for two reasons. First, in the United States policy makers are taking a second look at anti-trade commercial policy as a means to correct various imbalances and harms associated with trade. Moreover, the practical application of this tool may be easier, as the executive branch has broader authority over trade policy relative to changes in the tax code.

A second motive is that import tariffs and a progressive tax system may play similar social insurance roles. Early work by Corden (1974) and Baldwin (1982) speculate that one reason for tariffs is exactly to provide social insurance. Eaton and Grossman (1985) also show that a motive for a positive tariff is to provide social insurance. More starkly, Newbery and Stiglitz (1984), in a setting with risk and incomplete markets, provide an example that free trade is Pareto inferior to autarky.

To compare a progressive tax system to import tariffs, we perform the following quantitative exercises. Starting from our baseline economy, we compute and trace out social welfare under different mixes of tariffs and tax progressivity.

One important issue is what to do with tariff revenue. Like tax revenue, we treat this as pure waste. Furthermore, we turn off the idea that the tariff becomes a substitute for income tax as a revenue collection device. To avoid this issue, for a given level of progressivity, we hold constant average tax rates constant as we vary the tariff rate. So the way to think about the question is: “Given a labor income tax system, does a tariff improve welfare or not?”

Figures 9 and 10 illustrate the results. Each panel is an economy calibrated to a different level of openness—an import share of 20 percent and an import share of 40 percent and tax system with progressivity of 0.18. On the horizontal axis, we have the import tariff and the tax progressivity parameter. On the vertical axis, we report welfare in consumption-equivalent units relative to the baseline level of progressivity. The surface traces out welfare as we varied tariff rates and tax progressivity. The star indicates the optimal policy.
Figure 9: Welfare, Progressivity and Tariff Policy; Imports/GDP = 0.20 in Baseline

Figure 10: Welfare, Progressivity and Tariff Policy; Imports/GDP = 0.40 in Baseline
Figures 9 and 10 show that tariffs are not welfare-improving. For any level of progressivity, as tariffs increases lower welfare. Moreover, the worst policy mix is a regressive tax system with a high tariff. In contrast, the optimal policy is a zero tariff with a progressive tax system (with the optimal level of progressivity the same as shown in Table 2). To summarize, we find no evidence that an import tariff is a welfare improving approach to deal with the costs of opening to trade.

What are the distinguishing features between our work and say, Eaton and Grossman’s (1985) or Newbery and Stiglitz’s (1984). It’s hard to say, given the very stark differences in our model relative to their work. However, we speculate that there are some important differences in terms of policy instruments and insurance opportunities that might make a tariff or autarky look favorable. For example, Eaton and Grossman (1985) allow for lump-sum redistribution of tariff revenue and state-contingent tariffs, and abstract from redistribution via labor income taxes. Newbery and Stiglitz (1984) deliberately turn off any other mechanisms for households to self-insure (either through non-state-contingent assets or other margins such as migration).

One caveat to this analysis is that the tariff policy we consider is stark—one uniform tariff for all imported goods. In contrast, one could imagine situations exploring a more nuanced tariff structure. For example, a tariff policy that has a progressive nature which protects against the most directly exposed. We leave this for future research.

7. Conclusion

We motivated this paper by discussing concerns regarding globalization. And we quantitatively explored the idea that redistributive taxation could be used to provide social insurance and, in turn, mitigate the losses from trade that certain segments of the population have experienced.

Our results can be summarized as follows. First, the tension in our economy is between the misallocation of households across space versus the benefits of social insurance. This gave rise to an optimal degree of progressivity and we found that it should increase as the economy becomes more exposed to trade. Independent of weather the degree of progressivity is optimal or not, a progressive tax system becomes increasingly beneficial as the economy becomes open, enhancing the gains from trade. Finally, we find no evidence that an import tariff is an effective approach to dealing with the costs of trade.

Many open questions and directions for future research remain. Let us suggest two. First, we would prefer a tighter mapping from the evidence ofAutor, Dorn, and Hanson (2013) to normative welfare statements and policy conclusions. We are currently pursing such a strategy in our parallel work of Lyon and Waugh (2018). Second, the issues we explored in this paper relate to broader concerns about automation and technological change. Trade is a labor saving
technology. It benefits many, but hurts those directly exposed to it. As exposure to this labor saving technology increases, increasing progressivity improves welfare.
References


LYON, S., AND M. WAUGH (2018): “Quantifying the Losses from International Trade,”.


A. Connection with National Accounts

This section connects these equilibrium relationships to national income and product accounts (NIPA). This will help facilitate an understanding of the connection between trade imbalances and household’s consumption-savings decisions. Note that in all of this derivation, we normalize the price of the final good to one.

The Income Side of NIPA. It is first useful to start from an income side measure of production (or GDP) in our economy. Given competition, the value of aggregate production of the final good must equal aggregate payments for intermediate goods. The latter equals aggregate payments to labor in the production of all intermediate goods.

\[
Y = \int w(s) \mu(s) ds \tag{38}
\]

Examining (29) and (38) allows us to connect aggregate income with consumption. Specifically, by integrating over the consumers budget constraint, then noting how total value added must equal take home pay plus taxes/transfers from the government, one can substitute (38) into (29). Then from the definition of the governments budget constraint (which includes tax revenue and tariff revenue), we arrive at the following:

\[
Y = C + G - RA + A' - \int_a s f(s)\lambda(s,a) v(s) \mu(s) ds \tag{39}
\]

so aggregate income equals consumption (private and public) minus (i) returns on assets (ii) new purchases of assets (iii) plus moving costs (iv) minus tariff revenue. The last point follows from noting that government consumption is both tax revenue and tariff revenue.

This basically says that income/production must equal consumption net of income not associated with production (i.e. returns on assets and home production) plus “investment” in assets and moving costs. For example, if consumption is larger than income one reason is that (in aggregate) households (on net) are borrowing from abroad (\( A' < 0 \)).

The Production Side of NIPA. The value of aggregate production of the final good must equal the value of intermediate goods production.

\[
Y = \int s p(s) \mu(s) ds \tag{40}
\]

which we can then connect with the expenditure side of GDP through the market clearing conditions for intermediate goods and final goods. Specifically, by connecting the production side with the demand side for non-traded goods in (20), imports in (22) and exports in (24) and
the equating final demand with consumption we have

\begin{align*}
Y &= C + \tilde{G} + \int_{s} p(s) \text{exports}(s) ds - (1 - \tau_f) \int_{s} p(s) \text{imports}(s) ds \\
&= C + G + \int_{s} p(s) \text{exports}(s) ds - \int_{s} p(s) \text{imports}(s) ds. 
\end{align*}

Or GDP equals consumption (private and public) plus exports minus imports.

**Savings, Trade Imbalances, and Capital Flows.** Finally, we can connect the income side and the production side the national accounts to arrive at a relationship between asset holdings and trade imbalances. By working with both (39) and (42) we get the following relationship

\begin{align*}
Y - C - G &= \int_{s} p(s) \text{exports}(s) ds - \int_{s} p(s) \text{imports}(s) ds, \\
&= -rA + (A' - A) + \int_{a} \int_{s} m_{m}(s, a) \lambda(s, a) ds da - \tau_f \int_{s} p(s) \text{imports}(s) ds, 
\end{align*}

where $r$ is the net real interest rate. This relationship says the following: aggregate savings equals the trade imbalance. And this, in turn, we can connect the trade balance with the savings decisions of the households. That is the trade balance equals payments on net asset holdings plus net change in asset holdings (adjusted for moving costs and tariff revenue). To map this into Balance of Payments language: the trade imbalance plus foreign income payments is the current account; the capital account is the net change in foreign asset holdings; then (we suspect) moving costs would show up as the “balancing item.”

To see this, consider the special case where moving costs are zero and tariffs are zero. Then we have the relationship

\begin{align*}
Y - C - G &= \int_{s} p(s) \text{exports}(s) ds - \int_{s} p(s) \text{imports}(s) ds = -rA + (A' - A). 
\end{align*}

Here if exports are greater than imports, then this implies that the households in the home country are doing several things. The trade surplus may reflect that households (on net) are making debt payments ($rA$ is negative). Second, the trade surplus may reflect that the households (on net) are acquiring foreign assets ($A' - A$ is positive). Finally, note that in a stationary equilibrium, the trade imbalance only reflects payments from foreign asset holdings. This implies that the current account and capital account are always zero in a stationary equilibrium, but that trade may be imbalanced.
B. Computational Appendix

This section describes our computational approach to solving and calibrating the model. All materials are posted at https://github.com/mwaugh0328. The code is presented in two different languages: Matlab and Julia. The implementation in both languages follows the same core steps, but the details are slightly different. The discussion below follows the Matlab code and the instructions in the island_solve_progressive_NAG.m file. For a detailed explanation of how the Julia code works, see the file model/README_Julia.md.

- We approximate the continuous asset, productivity, and world price states by discretization. The asset space follows a non-uniform grid with grid points clustered near the borrowing constraint. The number of grid points was set to 50; the results are not sensitive to increases in this number. We use the method of Rouwenhorst to discretize the productivity and world price process. We use 10 states for productivity and world prices each, thus there are 100 different states $s$.

- Guess a proposed price function $\hat{p}(s)$ and government policy $\hat{\delta}$.

- Compute after-tax wages.

- Solve the households problem in (10). This is performed using value function iteration. One technique we use to facilitate finding a solution to the equilibrium is to “smooth” out the discrete choice problem. We do this by assuming that there are additive logistically distributed preference shocks with parameter $b_{\text{smth}}$ and these preference shocks are independently distributed across each choice. These enter into the choice problem in (10) by adding onto each option. What this gives rise to is, for each asset holdings state and state $s$, there will be a non-zero mass of households choosing all options. The probabilities take the familiar logit form. We tune the parameter $b_{\text{smth}}$ to ensure that it is small and not affecting the economics of the problem, but at the same time ensure that we find a solution.

  Smoothing in this manner is important as it facilitates the use derivative based solvers in finding an equilibrium $p(s)$. This in turn results in a dramatic speed-up in the computation of an equilibrium. See, e.g., Morten (2016) who employs a similar approach.

- Given the policy functions associated with the solution to the problem in (10), we compute the stationary distribution over assets and states $s$, i.e. $\lambda(a,s)$ which is of size 50 (for each asset state) and 100 for each state $s$. This process is sped up using sparse matrices, see the code island_invariant.m for details.

- Given the stationary distribution, we can compute excess demand functions for all islands $s$ which also must respect the inequalities implied by (21) and (23). These conditions imply
that the problem of finding a price function consistent with a stationary equilibrium can be represented as a mixed complementarity problem (see, e.g., Miranda and Fackler (2004)). To smooth out the nondifferentiability issues with the complementarity problem we pass the excess demand functions through Fischer’s function. Again, see Miranda and Fackler (2004).

- Update \( \hat{p}(s) \) and government policy \( \hat{\delta} \) and proceed until convergence criteria are met.

In solving for the equilibrium, we employed derivative based solvers. One solver that we found much success with is the \( \text{c05qc} \) solver from Numerical Algorithms Group. MATLAB’s \( \text{fsolve} \) with central finite differences performed well too.

Our calibration approach employed the following technique. Let \( \Theta \) be the parameter vector we chose to match some moments. In our specific case, this is \( \Theta = \{\tau, m, B\} \) or the trade cost, the moving cost, and the disutility of labor. We then jointly solved for \( \{p(s), \delta, \Theta\} \) in one step. That is we asked the algorithm described to find a price vector and set of parameters such that (i) equilibrium conditions are satisfied and (ii) model implied moments match our target empirical moments. This avoided a more standard, but time consuming approach of guessing a \( \Theta \), solving for an equilibrium, updating \( \Theta \), etc. Extensive sensitivity analysis found no issues surrounding multiplicity of equilibrium.