ABSTRACT

Evidence suggests that trade with China harmed the US economy in the 2000s—a rapid rise in import penetration, expanding trade deficits, and local-labor-market evidence on the negative impact of trade on wages and employment are all pieces of evidence pointing in this direction. We provide an interpretation of this evidence by developing a dynamic, standard incomplete market model with Ricardian trade and frictional labor markets. We discipline our model by calibrating it to match aggregate and local-labor-market evidence then study the transition dynamics of the economy in response to a “China Shock.” Consistent with , our model predicts import-competition-exposed workers experience (relative) wage losses and reductions in labor force participation. In aggregate, however, the China shock leads to a large increase in consumption and employment. So how much did the losers from trade lose? Preliminary analysis finds that those most exposed to trade suffer welfare losses of one percent in consumption equivalents.

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Trade generates winners and losers, but that the winners win more than the losers lose. This phrase is said often, but does little to assuage the concerns of people and policy makers about the forces of globalization—that the losses from trade are large; that there are insufficient mechanisms to insure against these losses; that globalization simply propagates existing inequality. This paper takes one step toward evaluating these concerns by answering the question: How much do the losers lose from trade?

A popular narrative hints that the losses from trade were large. Stating in early 2000s and continuing into the beginning of 2008, US imports as a fraction of GDP increased by seventy percent—Chinese imports accounted for all of this growth. At the same time, the US trade deficit as a fraction of GDP expanded by two percentage points. These facts suggest that the growth in Chinese import penetration did not arrive with equivalent export and employment opportunities for those displaced from trade. Autor, Dorn, and Hanson (2013) provides evidence to support this idea. Using evidence from differential changes across local labor markets, they find that increased exposure of a labor market to Chinese trade resulted in reductions in labor income and labor force participation.

In this paper, we provide an evaluation of this evidence by developing a dynamic, standard incomplete markets model with Ricardian trade and frictional labor markets. A quantitative, structural evaluation of this evidence is important for several reasons. First, there are well known issues about the aggregation of evidence that exploits differential changes in local labor markets (as in Autor, Dorn, and Hanson (2013)). Our model provides a structural interpretation of the evidence in Autor, Dorn, and Hanson (2013) and, hence, a mapping into aggregate conclusions and welfare statements.

A second issue is that the evidence in Autor, Dorn, and Hanson (2013) suggests that households are acting to mitigate the negative labor market impacts. For example, an interpretation of the decrease in labor force participation is that wages are too low relative to the value of leisure and, hence, trade-exposed households substitute into leisure. A key feature of our analysis is that we study a model in which households have multiple mechanisms to mitigate income shocks (some of which are trade related). In particular, households can self-insure, opt-out of the labor force, and/or migrate. These mechanisms lead to a tension between the negative labor market consequences from trade and the ability of households to mute these shocks.

A final issue relates to the trade deficit. The evidence of Autor, Dorn, and Hanson (2013) is particularly compelling given that the deteriorating trade deficit during the 2000s suggests employment opportunities for those displaced from trade did not come from increases in export opportunities. While in line with traditional trade theory, this perspective ignores the fact that a rising trade deficit corresponds with a welfare improving situation where households are increasing their consumption above and beyond their savings. In other words, the of China
came with an expansion of both intra- and inter-temporal trading opportunities. This later point is closely related to the “global savings glut” hypothesis put forth by Bernanke (2005). The explicit modeling of consumption-savings decisions of households allows us to entertain alternative perspectives on the trade imbalance.

Our model builds on existing neoclassical trade theory and then departs from it in important ways. As in the model of Eaton and Kortum (2002), there is a continuum of goods with competitive producers who are heterogenous in productivity; comparative advantage determines the pattern of trade. As in Lucas and Prescott (1974) the labor market is frictional and labor can only move across different goods producing markets (within a country) after paying some cost. Dynamics at the micro-level arise as good-level productivity stochastically evolves and labor and the pattern of trade adjust in response.

Labor market frictions delivers losses in labor income to import-competition-exposed workers very much consistent with the evidence in Autor, Dorn, and Hanson (2013). We show how our model delivers a structural interpretation of the regressions in ADH and a rationalization for their instrumental variable strategy. In particular, our model structurally relates goods-market-level wages with import exposure through the share of domestic production relative to consumption of that commodity and the elasticity of substitution across commodities. Thus, an increase in import exposure leads to a decline in goods-market-level wages.

We depart from recent quantitative trade theory by positing an environment where insurance markets are incomplete. As in the standard incomplete markets model (Huggett (1993), Aiyagari (1994)) households can self-insure by accumulating a non-state contingent asset.\footnote{The quantitative nature of our analysis distinguishes this paper from earlier discussions about risk, incomplete markets, and the gains from trade, e.g., see Newbery and Stiglitz (1984) and Eaton and Grossman (1985). Furthermore the standard incomplete markets setting take the reason that insurance markets are missing as given; Dixit (1987, 1989a,b) shows how certain conclusion depend upon the modeling as to why these markets are missing.} In contrast to essentially all models of trade and labor market dynamics, this aspect of the model allows for the partial—but not complete—pass through of income shocks into consumption. Given that risk sharing is often found to be incomplete (see, e.g., Cochrane (1991), Attanasio and Davis (1996)), this formulation provides a middle ground between a complete markets benchmark where the losses from trade are shared equally and where all households are hand-to-mouth with no opportunities to smooth out shocks.

In addition, we endow households with two additional margins to mitigate shocks. First, we allow households to migrate and move to alternative labor markets. This margin has been an important dimension in models of trade and labor market dynamics (see, e.g., Kambourov (2009), Artuç, Chaudhuri, and McLaren (2010), Dix-Carneiro (2014), Caliendo, Dvorkin, and Parro (2015)). Second, households can exit the labor force and substitute into leisure. Our formulation of labor supply follows the work of Rogerson (1988) and more closely Chang and
Kim (2007). This formulation of labor supply allows our model to speak directly to the evidence on labor force participation in Autor, Dorn, and Hanson (2013). Moreover, this formulation is consistent with the evidence discussed in Keane (2011)—our model has a small micro-labor supply elasticity, but can deliver a large aggregate labor supply elasticity due to movements along the extensive margin.

We discipline our model by calibrating it to match aggregate and cross-sectional evidence for the US and then study the transition dynamics of the economy in response to a “China Shock.” We focus on two types of shocks. First, we study a change in the ability to import goods. We think about this change as correspond with China’s accession to the World Trade Organization and the the policy changes and reduction in uncertainty surrounding it (see, e.g., Pierce and Schott (2016)). Second, we study a reduction in the world real interest rate. We think of this shock as reflecting a global savings glut shock echoing the arguments of Bernanke (2005). The idea is that an expansion of global savings from developing countries (including China) lead to a world wide decrease in real interest rates.

An important feature of our analysis is that we focus on transition paths. That is we change parameter values in an unanticipated fashion and compute the transition path from the initial stationary equilibrium to the new stationary equilibrium. Thus, our welfare gains/losses include the costs of adjustment in response to the shock.

In our baseline specification with a reduction in trade costs and a reduction in the world interest rate— trade expands and there is a widening trade imbalance. In our quantitative exercise, the trade imbalance is not target and declines by 0.75 percentage points. This is nearly half of the actual change in trade deficit seen in the subsequent years after China’s accession to the WTO and large expansion of trade with the US. This sequence of events gives rise to a boom in output, consumption, and labor supply.

We then compare evaluate the micro-level implications of our model to the evidence of Autor, Dorn, and Hanson (2013). We do this by constructing model generated data in a fashion consistent with the empirical analogs. And then we implement the same IV strategy they employ on our model generated data. Despite the boom occurring in the aggregate economy, we find trade exposure resulting in large wage losses, reductions in labor supply. The magnitudes are in the ball park, but different than those of Autor, Dorn, and Hanson (2013) and we discuss possible reasons for this.

We then compute welfare. First, average welfare gains from the China Shock are our 0.60 percent in lifetime consumption equivalents. There is, however, large dispersion around this number. On average, initially import exposed regions in our economy experience losses of 0.80 percent (with some losing substantially more).

Finally, a key issue that comes out of this analysis is the trade imbalance. In our baseline
specification, the China shock is composed of two effects, a reduction to the cost of importing goods and a reduction in the world interest rate. Consistent with the data, this gave rise to a deteriorating trade imbalance. In contrast, only the cost of importing goods is changed, the model tells a different story. The trade imbalance improves, consumption declines, labor supply declines. And, surprisingly, the welfare gains from trade are on average smaller and the losers from trade lose more than when the trade imbalance behave more like the data.

**Related Literature.** Our modeling framework is related, but distinct from an exciting and growing body work on trade and labor market dynamics (see, e.g., Kambourov (2009), Artuç, Chaudhuri, and McLaren (2010), Dix-Carneiro (2014), Caliendo, Dvorkin, and Parro (2015), Coşar, Guner, and Tybout (2016)). We depart from this literature by studying an economy in which households face labor income shocks and incomplete markets. The cost of this departure is that we are unable to incorporate the the geographic and sectoral detail found in this work (see, e.g., Caliendo, Dvorkin, and Parro (2015)) due to computational complexities.

We view this as an important departure for several reasons. First, it because it breaks the link between labor market outcomes and welfare. And it breaks the link in non-obvious ways. For example, if households are sufficiently wealthy, they can smooth out the trade induced losses to labor income and, thus, the losses from trade are small. This is an example where households do have a mechanism to mitigate the losses from trade through self-insurance. However, if workers in import-competing sectors are already wealth poor, then the welfare losses from trade could be quite large. This is an example where the previously poor bear the burden of globalization and inequality is amplified.

Second, it opens the door to questions about government policy and the correct response when the economy is increasingly exposed to trade. As we show in our parallel work in Lyon and Waugh (2018b), we find that progressivity should increase with openness to trade and that progressivity is an important tool to mitigate the negative consequences of globalization.

### 1. Motivating Facts

There is a body of evidence suggesting that trade with China harmed the US economy in the 2000s. This section discusses this evidence and it provides benchmarks that we will ask our quantitative experiment to replicate.

#### 1.1. Macro Facts

**Fact 1: Large rise in US Import Exposure.** Figure 1 plots US imports of goods and services divided by GDP for the time period 1990 until the beginning of 2008. The red line is US imports in total, the blue line is all but China, thus the difference reflects Chinese imports relative to GDP. US import exposure grew dramatically during this time period from approximately 10
percent of GDP to near 17 percent of GDP by the beginning of 2008.

The blue line shows that non-Chinese US import exposure was roughly constant over this time at around 10 percent. What this implies that nearly all the rise in US import exposure during this time period came from an increase in Chinese import exposure. Moreover, most of this expansion was after China’s accession to the World Trade Organization at the end of 2001.

**Fact 2: An Expansion in the US Trade Deficit.** Figure 2 plots US exports minus imports divided by GDP for the time period 1990 until the beginning of 2008. During this time period, the trade deficit grew by about five percentage points in total; three percentage points prior to the large expansion in Chinese import expansion, and then two percentage points subsequently.

From a trade perspective, these two facts suggest that the “traditional” replacement of job opportunities did not occur with the rise in Chinese trade. That is import exposure was not offset by an increase export opportunities and, thus, was a drag on the US labor market in the 2000s as the facts below suggest. An alternative perspectives on Figure 2 is the “global savings glut” hypothesis put forth by Bernanke (2005). Developments in international credit markets during the 1990s and 2000s lead to an glut of foreign savings (some of which is Chinese related) and the US borrowed these funds and led to an expansion of the trade deficit.

### 1.2. Micro Facts

The next three facts focus on labor market outcomes from Autor, Dorn, and Hanson (2013). The idea behind Autor, Dorn, and Hanson (2013) is to exploit changes in the variation in trade exposure at the commuting zone level (see Tolbert and Sizer (1996)) and correlate it with changes in labor market outcomes such as earnings and labor force participation. Their measure of trade exposure is:

\[
\Delta IPW_{uit} = \sum_j \left( \frac{L_{ijt}}{L_{it}} \right) \left( \frac{\Delta M_{ucjt}}{L_{ijt}} \right)
\]  

(1)

where \( u \) stands for united states, \( c \) stands for China, \( i \) is a commute zone, \( t \) is time, and \( j \) is industry. This measure essentially takes aggregate US imports from China \( M_{ucjt} \) for industry \( j \) and essentially apportions these imports to a commute zone based on that commute zones share of national employment in that industry. It then aggregates across industries for that commute zone.

Given this measure of trade exposure, they estimate its affect on labor market outcomes with the following empirical specification:

\[
\Delta L_{it} = \gamma_t + \beta \Delta IPW_{uit} + \text{controls}_i + \epsilon_{it}.
\]  

(2)
Figure 1: US Imports

Figure 2: US Trade Deficit
A key issue is that the error term may incorporate factors that are simultaneously changing a commute zone’s trade exposure and labor market outcomes. As our model makes clear (see Section 4.3), local productivity shocks would be a key threat to identification, i.e. they would both change the pattern of trade and labor market outcomes. To avoid this problem, Autor, Dorn, and Hanson (2013) estimate (2) using an other countries imports from China as an instrument.

In the facts below, we report estimates after standardizing the $\Delta IPW_{uit}$. That is we demeaned this measure and divided by its standard deviation. As in Autor, Dorn, and Hanson (2013) all labor market outcomes are converted to 10 year changes.

**Fact 3: Import Exposure Decreased Household Income** The first column in Table 1 reports Autor, Dorn, and Hanson’s (2013) estimate of the response of commute zone household level income to trade exposure. What this coefficient means is that a one standard deviation increase in trade exposure reduced wage growth by four percent over 10 years. To put this in context, average wage growth over the period 2000-2007 was only about six percent (converted to 10 year change). In other words, these estimates suggest that moderate exposure to trade eliminates 2/3 of expected wage growth.

**Fact 4: Import Exposure Increased Non Participation** The second column in Table 1 reports the response of commute zone nonparticipation in the labor force in response to trade exposure. The magnitude of this coefficient means is that a one standard deviation increase in trade exposure reduced participation in the labor market by 1.11 percentage points. Average non-participation across commute zones, across all time periods was about 25 percent.

**Fact 5: Muted Migration Response.** The first column in Table 2 reports the estimate of the response of commute zone population to trade exposure. What this coefficient means is that a one standard deviation increase in trade exposure reduced a commute zone’s population by
Table 2: ADH and GLM Evidence: Migration and Trade Exposure

<table>
<thead>
<tr>
<th></th>
<th>ADH ∆ Population</th>
<th>GLM, ∆ Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standardized ∆ IPW</td>
<td>−0.05</td>
<td>−1.43</td>
</tr>
<tr>
<td></td>
<td>[−1.51, 1.41]</td>
<td>[−3.33, 0.48]</td>
</tr>
</tbody>
</table>

Note: Values in brackets report 95-5 confidence intervals. Variable definitions are as follows. ∆ Population corresponds with the log change in population. ∆ IPW is standardized by netting out the mean and dividing by the standard deviation. Greenland, Lopresti, and McHenry (2017) (GLM) replace ADH regional controls with agged population growth at the commute zone level.

−0.05 log points, moreover this effect is not statistically different than zero. This suggests that trade exposure did not induce households to reallocate across labor markets as standard trade theory would predict.

Greenland, Lopresti, and McHenry (2017), however, call into question this particular finding of ADH. For our purposes, an important issue is the treatment of pre-trends of population growth in commute zones. In the ADH specification, regional controls (e.g. midwest, pacific northwest, etc) are included and thus, control in broad terms for different population trends across regions. Greenland, Lopresti, and McHenry (2017) suggest instead using lagged population growth at the commute zone level. Visual inspection of pre-trends and regression evidence suggest pre-trends at the commute zone are an issue.

The second column of Table 2 reports the effects when reginal controls are replaced with lagged population. Here the the point estimate increases by an order of magnitude, e.g. from essentially zero to -1.40 log points. That is a one standard deviation increase in trade exposure reduces a commute zones population by almost one and a half log points. The dramatic change in point estimates suggest that prior to the ADH window of 1990, commute zones that would eventually experience growth in trade exposure, were experiencing population growth in the 1980s. Not taking this fact into account biased the ADH estimates upward. With that said, at conventional levels of statistical significance, the point estimate is not statistically different from zero.

Greenland, Lopresti, and McHenry (2017) raise two other issues that lead to finding out migration from trade exposure that lead to similar findings Table 2. One issue is the use of Census data versus nationally representative samples, use of Census data delivers similar point estimates in Table 2 but stronger and significant effects for the young vs. old. A second issue is the time window, extending the window to 2010 leads to statistically significant effects with point
estimates comparable to Table 2. In sum, relative to the body of evidence provided by Greenland, Lopresti, and McHenry (2017), ADH’s specification is the only specification that finds no migration response.

1.3. Discussion

Taken together, these facts suggest a compelling narrative: At the macro level, there was a large increase in import exposure from China and no corresponding increase in export opportunities. And at the micro level, the evidence suggests this wreaked havoc on labor market in the United States—households absorbed losses in labor income, stopped participating in the labor market, and with some migration from trade exposed regions.

How to interpret this evidence? First, there is an issue about the interpretation of the cross-sectional estimates in (1) and how to map them into aggregate conclusions. The model that we develop below plays an important role here. And as we describe in Section 4 the model bridges this gap, clarifies what these estimates are informative about, and provide key inputs into our qualitative analysis.

Second, a model is needed to jump from changes in income and non-participation to statements about welfare. Our presumption is that while households have limited access to direct insurance markets, they do have a myriad of ways to smooth out negative labor market outcomes. One is self-insurance, another is that leisure is a substitute for leaving the labor market, a third is migration. Thus, we develop a model that allows us to entertain multiple mechanisms for which households can mitigate the negative outcomes that may arise from international trade.

A final issue relates to the trade deficit. Traditional trade theory’s perspective on this suggests that increases in import penetration from China did not come with increases in export and employment opportunities for those displaced from trade. This perspective ignores the idea that a rising trade deficit implies households are increasing their consumption above and beyond their savings. In other words, the of China came with an expansion of both intra- and inter-temporal trading opportunities.

2. Model

Here, we describe a model of international trade with households facing incomplete markets and frictions to move across labor markets. The first subsection discusses the production structure; the second subsection discusses the households.

Below, since we focus on the perspective of one country, country subscripts are omitted unless necessary. Similarly, time subscripts are omitted unless necessary.
2.1. Production

The model has an intermediate-goods sector and a final good sector that aggregates the intermediate goods. Within a country, there is a continuum of intermediate goods indexed by \( \omega \in [0, 1] \). As in the Ricardian model of Dornbusch, Fischer, and Samuelson (1977) and Eaton and Kortum (2002), intermediate goods are not nationally differentiated. Thus, intermediate \( \omega \) produced in one country is a perfect substitute for the same intermediate \( \omega \) produced by another country.

Competitive firms produce intermediate goods with linear production technologies,

\[
q(\omega) = z(\omega)\ell,
\]

where \( z \) is the productivity level of firms and \( \ell \) is the number of efficiency units of labor. Intermediate goods productivity evolves stochastically according to an AR(1) process in logs

\[
\log z_{t+1} = \phi \log z_t + \epsilon_{t+1},
\]

where \( \epsilon_{t+1} \) is distributed normally with mean zero and standard deviation \( \sigma_\epsilon \). The innovation \( \epsilon_{t+1} \) is independent across time, goods, and countries.

Firms producing variety \( \omega \) face competitive product and labor markets with households that supply labor elastically. Competition implies that a household choosing to work in market \( \omega \) earns the value of its marginal product of labor, which is the price of the good times the firm’s productivity \( z \).

Transporting intermediate goods across countries is costly. Specifically, consumers and firms face iceberg trade costs when importing and exporting their products. We allow for the import and export cost to differ with \( \tau_{im} > 1 \) being the cost to import a good from abroad and \( \tau_{ex} > 1 \) being the cost an export faces to ship goods onto the world market.

Intermediate goods are aggregated by a competitive final-goods produce who has a standard CES production function:

\[
Q = \left[ \int_0^1 q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}},
\]

where \( q(\omega) \) is the quantity of individual intermediate goods \( \omega \) demanded by the final-goods firm, and \( \rho \) controls the elasticity of substitution across variety, which is \( \theta = \frac{1}{1-\rho} \).
2.2. Households

Within a country, there is a continuum of infinitesimally small households of mass \( L \). Each household is infinitely lived and maximizes expected discounted utility

\[
E \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t) - B \frac{h_t^{1-\gamma}}{1-\gamma} + \nu_i^t \right\},
\]

(6)

where \( E \) is the expectation operator and \( \beta \) is the subjective discount factor. Period utility depends on both consumption of the final good, the disutility of labor, and a preference shock \( \nu_i^t \). The preference shock \( \nu_i^t \) is index by \( i \) which corresponds with choice of the household to move or not. As in Artuç, Chaudhuri, and McLaren (2010) and Caliendo, Dvorkin, and Parro (2015), this preference shock is independently and identically distributed across time and is distributed Type 1 extreme value distribution with scale parameter \( \sigma_{\nu} \).

Households live and work along the same dimension as the intermediate goods. That is, a household’s location is given by \( \omega \)—the intermediate goods sector in which it can work. Given their current location, households can choose to work, to move and work someplace else in the future, and to accumulate a non-state contingent asset. Below, we describe each of these choices in detail.

Working is a discrete choice between zero hours and \( \bar{h} \). Thus, the labor supply is purely on the extensive margin. If a household works, it receives income from employment in the intermediate-goods sector in which the household resides. In the following presentation, we normalize the value \( \bar{h} \) equal to one. If a household does not work, if receives home production \( w_h \). The value of home production partially determines the value of being out of the labor force and hence, the elasticity of labor supply on the extensive margin.

Households can move to an alternative intermediate-goods sector \( \omega' \) at some cost. Paying \( m \) in units of the final good allows the household to change where it can work in next period. We assume that the new location is a random labor market. Moving also triggers the realization of the \( \nu \) preference shock associated with the move.

Households residing in a intermediate-goods location face labor income risk associated with fluctuations in local productivity and fluctuations in world prices. We do not allow for any insurance markets against this risk, but let households accumulate a non-state contingent asset \( a \) that pays gross return \( R \). We treat \( R \) as exogenous and not solved for in equilibrium. An interpretation is that this country faces a large supply of assets at this rate. Households face a lower bound on asset holding \( -\bar{a} \), so agents can acquire debt up to the value \( \bar{a} \).

**State Variables.** The individual state variable of a household are its location, asset holdings \( a \), and preference shocks \( \nu^i \). The island-level state variable is the domestic productivity state
and world price state. The aggregate state is a distribution over island-level state variables and asset holdings.

Let us expand on this a bit more. The wage per efficiency unit that a household receives is an important island-level object impacting individual decisions. The wage per efficiency unit depends on the value of the marginal product of labor on that island. The marginal product depends on a country’s productivity level. The “value” part depends on (i) the world price and (ii) the labor supply decisions of households residing on the island. Given our preference specification in (6), households’ labor supply decisions depend on the distribution of asset holdings within the island. Thus, this is where the aggregate state matters for island-level outcomes.

The presentation below depicts a stationary equilibrium. That is, the aggregate state—the distribution over island level states and assets holdings—is constant. Thus, to conserve on notation, we only carry around the households specific state variables: its own asset holdings, preference shocks, and island-level state variables associated with its location. In particular, let \( s \) denote the domestic productivity and world price combination associated with that island. Furthermore, because the CES aggregator is symmetric over varieties, it is sufficient to index islands by their productivity and world price state. The wage per efficiency unit a household earns is \( w(s) \).

**Budget Constraints.** Given the description of the environment, the budget constraints are as follows. For households that are working, the household’s period \( t \) budget constraint (all denominated in units of the final good) is

\[
a_{t+1} + c_t + \iota_{m,t} m \leq Ra_t + w_t(s),
\]

where the left-hand side are expenditures on new assets, consumption, and possibly moving costs with \( \iota_{m,t} \) being an indicator function equaling one if a household moves and zero otherwise. The right-hand side are income payments from asset returns and labor income.

If a household is not working, then the budget constraint is modified to ensure that home production is not used to accumulate assets or pay for moving costs. In this case, a non-working households budget constraint is

\[
c_t \leq w_h + |Ra_t - a_{t+1} - \iota_{m,t} m|^{+},
\]

where the right-hand side is home production plus any positive income from net asset holdings, net of moving costs. What this formulation insures is the home prosecution is just that. It can not be used for purchases in the market of assets or moving costs.

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\(^2\)Given an island with state \( s \), denote the measure of agents with asset holdings \( a \) as \( \lambda(s, a) \). Stationarity implies that this value is constant.
Recursive Formulation. The recursive formulation of the household’s problem is

\[ V(a, s, \nu) = \max \left[ V^{s,w}(a, s, \nu), V^{s,nw}(a, s, \nu), V^{m,w}(a, s, \nu), V^{m,nw}(a, s, \nu) \right]. \tag{9} \]

that is a discrete choice among four options: the value of staying and working; the value of staying and not working; the value of moving and working; the value of moving and not working. Unpacking each of these four options is the following. The value of staying and working is

\[ V^{s,w}(a, s, \nu) = \max_{a' \geq -\bar{a}} \left[ u(Ra + w(s) - a') - B + \nu^s + \beta EV(a', s', \nu') \right], \tag{10} \]

where \( u \) is the utility value over consumption and \( \nu^s \) is the preference shock associated with staying in its current location. The value of staying and not working is

\[ V^{s,nw}(a, s, \nu) = \max_{a' \geq -\bar{a}} \left[ u(w_h + |Ra - a'|^{\gamma}) + \nu^s + \beta EV(a', s', \nu') \right]. \tag{11} \]

The value of moving and working is

\[ V^{m,w}(a, s, \nu) = \max_{a' \geq -\bar{a}} \left[ u(Ra + w(s) - a' - m) - B + \nu^m + \beta V^m(a') \right], \tag{12} \]

where there are several key distinctions relative to (10). First, the moving cost, \( m \) is paid. Second, \( \nu^m \) is the preference shock associated with moving. Third, the continuation value is \( V^m(a') \) or the value associated with a move. Finally, the value of moving and not working is

\[ V^{m,nw}(a, s, \nu) = \max_{a' \geq -\bar{a}} \left[ u(w_h + |Ra - a' - m|^{\gamma}) + \nu^m + \beta V^m(a') \right]. \tag{13} \]

3. Equilibrium

We close the model by focusing on a small open economy equilibrium. The small open economy assumption is that there is no feedback from home country actions into world prices.\(^3\)

World Prices. World prices for commodity \( \omega \) evolve according to an AR(1) process in logs:

\[ \log p_w(\omega)_{t+1} = \phi \log p_w(\omega)_t + \epsilon(\omega)_{t+1}, \tag{14} \]

where \( \epsilon(\omega)_t \) is distributed normally with mean zero and standard deviation \( \sigma_w \) and is independent of the innovation to the home country’s productivity \( \epsilon_t \). We express these prices in units

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\(^3\)Relative to the trade and labor market dynamics literature, this is similar to the second specification solved in Artuc, Chaudhuri, and McLaren (2010). Moreover, it has the advantage (say, relative to Caliendo, Dvorkin, and Parro (2015)) of being relatively simple, yet allows us to specific about the interaction between trade flows and capital flows.
of the numeraire, which we take to be the final good in the home country.

**A Note on Notation.** We denote \( \pi(s) \) as the stationary distribution of productivity states and world prices induced by (4) and (14). And denote \( \mu(s) \) as the measure of households working on an island with state \( s \). This value is defined later in (27) and integrates over the labor supply choice of households (which depends upon their individual states and preference shocks).

### 3.1. Production Side of the Economy

Below, we describe the equilibrium conditions associated with the production side of the economy. These take as given the choices of the household.

**Final Goods Production.** The final-goods producer’s problem is:

\[
\max_{q(s)} P_h Q - \int p(s) q(s) \pi(s) ds,
\]

which gives rise to the following the demand curve for an individual variety:

\[
q(s) = \left( \frac{p(s)}{P_h} \right)^{-\theta} Q.
\]

where \( Q \) is the aggregate demand for the final good; \( P_h \) is the price associated with the final good which will be carried around briefly, but is ultimately normalized to the value one.

**Intermediate Goods Production.** The intermediate-goods-producer’s problem is

\[
\max_{q(s), \ell(s)} p(s) q(s) - w(s) \ell(s)
\]

or to choose the quantity produced to maximize profits. Competition implies that the wage per efficiency unit (in units of the final good) at which a firm hires labor is:

\[
w(s) = p(s) z
\]

or the value of the marginal product of labor. Only at the wage in (18) are intermediate-goods producers willing to produce.

**Intermediate Goods, International Trade, and Market Clearing.** To formulate the pattern of trade, we denote the set of prevailing prices that the final-goods producer in the home country faces as \( p(s), \tau_{im} p_w, p_w / \tau_{ex} \). The final-goods producer purchases intermediate goods from the low-cost supplier. This decision gives rise to three cases with three different market-clearing conditions: if the good is non-traded; if the good is imported; and if the good is exported.\(^4\)

\(^4\)This is more nuanced than the standard formulation in Eaton and Kortum (2002) due to the frictional labor...
Below, we describe demand and production in each of these cases.

- **Non-traded.** If the good is non-traded, then the domestic price for the home country must satisfy the following inequality: \( \frac{p_w}{\tau_{ex}} < p(s) < \tau_{im}\pi_w \). That is, from the home country’s perspective, it is optimal to source the good domestically and not optimal for the home country to export the product.

In this case, the market-clearing condition is:

\[
\left( \frac{p(s)}{\pi(s)} \right)^\theta Q = z (\mu(s)/\pi(s))
\]

or that domestic demand equals production. The left-hand part is demand and the right-hand side is supply. That is the productivity of domestic suppliers multiplied by the supply of labor units in that market.

- **Imported.** If the good is imported, then the domestic price for the home country must be \( p(s) = \tau_{im}\pi_w \). Why? If the price were lower, then it would not be imported. If the domestic price were higher, then the good will be imported with not domestic production and, thus, the prevailing domestic price will equal the imported price. With frictional labor markets, there may be some domestic production so the quantity of imports is

\[
\left( \frac{\tau_{im}\pi_w}{\pi(s)} \right)^\theta Q - z (\mu(s)/\pi(s)) > 0.
\]

That is home demand (net of home production) is met by imports of the commodity. Rearranging gives

\[
\left( \frac{\tau_{im}\pi_w}{\pi(s)} \right)^\theta Q = z (\mu(s)/\pi(s)) + \text{imports}(s)
\]

or domestic demand equals domestic production plus imports.

- **Exported.** If the good is exported, then the prevailing price must be \( p(s)\tau_{ex} = p_w \). Why? If the home price were larger, then the good would not be purchased on the world market. And the price can not be lower, as arbitrage implies that the price of the exported good sold in the world market must equal the prevailing price in that market. Finally, note that
only the trade cost, not the tariff, matters here. At this price, the quantity of exports is

\[
\left( \frac{p_w/\tau_{ex}}{P_h} \right)^{-\theta} Q - z (\mu(s)/\pi(s)) < 0
\]

(22)

- exports

or domestic demand net of production which should be negative, implying that the country is an exporter. Rearranging gives

\[
\left( \frac{p_w/\tau_{ex}}{P_h} \right)^{-\theta} Q = z (\mu(s)/\pi(s)) - \text{exports}(s)
\]

(23)

or domestic demand equals domestic production minus exports.

**The Final Good and Market Clearing.** The final good’s producer sells the final good to consumers. Thus, we have the following market-clearing condition

\[
Q = C = \int \int \int c(s, a, \nu) \lambda(s, a, \nu) d\nu da ds,
\]

(24)

where \(c(s, a, \nu)\) is the consumption policy function that satisfies the households’ problem, and \(\lambda(s, a, \nu)\) is the mass of consumers with state \(s\), asset holding \(a\), and preference shock \(\nu\) (defined below in (??)). This relationship says that household-level consumption—aggregated across all households—must equal the aggregate production of the final good \(Q\).

Market-clearing conditions for the intermediate goods in (19),(21), (23) and the aggregate final good in (24) summarize the equilibrium relationship on the production side of the economy.

### 3.2. Household Side of the Economy

The households in the economy make choices about where to reside, how much to work, and how much to consume. Here, we describe the equilibrium conditions associated with these choices. In the discussion below, we define the following functions—\(\{\iota_m(s, a, \nu), \iota_n(s, a, \nu), g_a(s, a, \nu)\}\)—as the move, work, and asset policy functions that satisfy the households’ problem in (9).

**The distribution of households across states.** We define the probability distribution of households across assets and states as \(\lambda(s, a, \nu)\). Furthermore, define the probability distribution of households in the next period as \(\lambda'(s', a', \nu')\). The distribution of households evolves across time according to the following law of motion:

\[
\lambda'(s', a', \nu') = \phi(\nu') \int \int \lambda(s, a, \nu)(1 - \iota_m(s, a, \nu))\pi(s', s) + \lambda(s, a, \nu)\iota_m(s, a, \nu)\pi(s') da d\nu ds.
\]

(25)
Equation (25) says that in the next period, the mass of households with asset holding $a'$ in state $s'$ with preference shock $\nu'$ equals several terms. First, the probability that preference shocks $\nu'$ are realized where $\phi$ is the probability density function associated with the Type 1 extreme value distribution. The second term is the mass of household that do not move multiplied by the transition probability that $s$ transits to $s'$—this is the first term in equation (25). The third term is the mass of households that do move, multiplied by the probability that they end up in state $s'$—this is the second term in equation (25). The probability, $\bar{\pi}(s')$, is given by the moving protocol—i.e., random assignment across islands according to the invariant distribution associated with $\pi(s', s)$. All of this is conditional on those households that choose asset holdings equal to $a'$, this is denoted by the conditionality under the innermost integral sign. And integrated across preference shocks and current island state $s$.

**Population and Labor Supply.** Given a distribution of households, we define the population of households on islands with state $s$ as

$$\int_{\nu} \int_{a} \lambda(s, a, \nu) da \, d\nu.$$  

(26)

In words, the population is found by just integrating over the mass across asset holdings and preference shocks.\(^5\) Then the supply of labor to intermediate good producers with productivity state $s$ is,

$$\mu(s) = \int_{\nu} \int_{a} \iota_n(s, a, \nu) \lambda(s, a, \nu) da \, d\nu,$$  

(27)

which is the size of the population residing in that market multiplied by the labor supply policy function and integrated over all asset states. This, then, connects the supply of labor with production in (19)-(23).

**Asset Holdings.** The distribution of asset holdings and consumption take the following form. Next period, aggregate net-asset holdings are

$$A' = \int_{\nu} \int_{a} \int_{s} g_{a}(s, a, \nu) \lambda(s, a, \nu) ds \, da \, d\nu,$$  

(28)

where $g_{a}(s, a, \nu)$ is the policy function describing asset holdings tomorrow, given the states today. A couple of points about this are warranted. First, this is in aggregate—some households may have positive holdings, while others may have negative holdings. Second, net asset holdings must always be claims on foreign assets since there is no domestic asset in positive supply

---

\(^5\)This calculation should be distinguished by the population on an individual island. This is latter value is found by dividing 26 by the measure of that island type.
3.3. A Stationary Small Open Economy (SSOE) Equilibrium

Given the equilibrium conditions from the production and household side of the economy, we define a “Stationary Small Open Economy (SSOE) Equilibrium” equilibrium.

A Stationary Small Open Economy (SSOE) Equilibrium. Given world prices \( \{p_w, R\} \), a stationary Small Open Economy Equilibrium is domestic prices \( \{p(s)\} \), policy functions \( \{ g_a(s, a, \nu), \iota_n(s, a, \nu), \iota_m(s, a, \nu) \} \), and a probability distribution \( \lambda(s, a, \nu) \) such that

i. Firms maximize profits, (15) and (17);

ii. The policy functions solve the household’s optimization problem in (9);

iii. Demand for the final and intermediate goods equals production, (19), (20), (22) and (24);

iv. The probability distribution \( \lambda(s, a, \nu) \) is a stationary distribution associated with \( \{ g_a(s, a, \nu), \iota_m(s, a, \nu), \pi(s', s, \nu) \} \). That is, it satisfies

\[
\lambda(s', a', \nu') = \phi(\nu') \int \int \int \lambda(s, a, \nu)(1 - \iota_m(s, a, \nu))\pi(s', s) + \lambda(s, a, \nu)\iota_m(s, a, \nu)\pi(s') \, da \, d\nu \, ds. 
\]

The idea behind the equilibrium definition is the following. The first bullet point (i) gives rise to the equilibrium conditions for the demand of intermediate goods in (16) and wages (18) at which firms are willing to produce. The second bullet point (ii) says that households are optimizing.

At a superficial level, bullet (iii) says that demand must equal supply. It’s meaning, however, deeper. The households’ choices of the matter for both the demand and the supply side. Specifically, it requires that prices (and, hence, wages) must induce a pattern of (i) consumption and (ii) labor supply such that demand for goods equals the production of goods.

Bullet point (iv) requires stationarity. Specifically, the distribution of households across productivity and asset states is not changing. Mathematically, this means that distribution \( \lambda(s, a, \nu) \) must be such that when plugged into the law of motion in (25), the same distribution is returned.

Finally, note that there is no requirement that the asset market clears—i.e., that (28) equals zero. This is an aspect of the small open economy assumption. At the given world interest rate \( R \), the assets need not be in zero net supply. This implies that trade need not balance, as the trade imbalance will reflect asset income on foreign assets and the acquisition of assets. After
adjusting for moving costs, this implies that the current account and capital account are always zero in a stationary equilibrium, but that trade may be imbalanced.

**Computation.** Computing a stationary equilibrium for this economy deserves some discussion. First, this economy is unlike standard incomplete markets models in which only one or two prices (e.g., one wage per efficiency unit and/or the real interest rate) must be solved for. In contrast, we must solve for an equilibrium function $p(s)$. Thus, the iterative procedure is to (i) guess a price function; (ii) solve the household’s dynamic optimization problem; (iii) construct the stationary distribution $\lambda(s, a, \nu)$; (iv) check whether markets clear; and (v) update the price function. See, e.g., Krusell, Mukoyama, and Şahin (2010), who solve a similar problem.

Second, while the household’s problem contains three state variables, the i.i.d. assumption on $\nu$ allows us to integrate out the preference shock, compute choice probabilities, and then work directly on the distribution across islands and asset states, i.e. $\lambda(s, a)$.$^6$ The Type 1 extreme value distribution allows us to perform this integration step in closed form. In addition to allowing our model to match certain targets, these preference shocks also make the aggregate economy continuous in its price (modulo the discussion below) and parameter space which facilitates the quick computation of solutions to a stationary equilibrium.

Third, an important observation is that the inequalities in (20) and (22) impose additional structure on an equilibrium. The observation is that when domestic demand and supply are not equal, the price in those markets must respect bounds on international arbitrage. This implies that the problem of finding a price function consistent with a stationary equilibrium can be represented as a mixed complementarity problem (see, e.g., Miranda and Fackler (2004)). Appendix B provides a complete description of our solution procedure and links to our code repository.

4. Model Properties

This section describes some qualitative properties of the model. Below we focus on three issues (i) the pattern of trade across labor markets (ii) how trade exposure affects wages and how our model relates to the empirical approach/specification of Autor, Dorn, and Hanson (2013). Finally, we use these results to motivate our quantitative exercise.

$^6$To see this point, simply examine (25) and first integrate out the $\nu$'s. Integrating the individual policy functions over the $\nu$'s are the choice probabilities of a particular action, given state $s, a$, which take the familiar logit form given the distributional assumption.
4.1. Trade

To illustrate the pattern of trade across islands, first define the following statistic:

\[ \omega(s) := \frac{p(s)z\mu(s)}{p(s)z\mu(s) + p(s)\text{imports}(s) - p(s)\text{exports}(s)}. \]  

(30)

What does equation (30) represent? The denominator is the value of domestic consumption: everything domestically produced plus imports minus exports. The numerator is production. The interpretation of (30) is how much of domestic consumption at the island/variety level the home country is producing. This is similar to the micro-level “home share” summary statistic emphasized in Arkolakis, Costinot, and Rodríguez-Clare (2012). As we discuss below, this statistic (i) provides a clean interpretation of a labor market’s exposure to trade and (ii) is tightly connected with local labor market wages.

Figure 3 plots the home share (raised to the power of inverse \( \theta \)) by world price and home productivity. There are three regions to take note of: where goods are imported, exported, and non-traded. First, in the regions where the home share lies below one, demand is greater than supply, and, hence, goods are being imported. This region naturally corresponds to the situation with low world prices or low home productivity—i.e. the economy has a comparative disadvantage in producing these commodities.

Second, in the regions where the home share lies above one, supply is greater than demand, and, hence, goods are being exported. This region corresponds to high world prices or high home productivity. In other words, this is where the country has a comparative advantage and is an exporter of the commodities.

Third, there is the “table top” region in the middle, where the home share equals one. Hence, this is the region where the goods are non-traded. Exactly like the inner, non-traded region in the Ricardian model of Dornbusch, Fischer, and Samuelson (1977), the reason is trade costs. In this region, world prices and domestic productivity are not high enough for a producer to be an exporter of these commodities given trade costs. Furthermore, world prices and domestic productivity are not low enough to merit importing these commodities either. Thus, these goods are non-traded.

Finally, unlike Dornbusch, Fischer, and Samuelson (1977) or Eaton and Kortum (2002), it is important to reflect on the stochastic nature of this economy. While the stationary equilibrium of the economy leads to the stationary pattern of trade seen in Figure 3, individual islands transit between different states (world prices and domestic productivity). For example, an island may be an exporter, but given a sequence of bad productivity shocks, the island will stop exporting and maybe even become an importer of a commodity it once exported.
Figure 3: Trade: Home Share, $\omega(s)^{\frac{1}{\nu}}$

Figure 4: Wages
4.2. Trade and Wages

One can connect the pattern of trade across islands/labor markets in Figure (3) with the structure of wages in the economy. As we show in the Appendix and in Lyon and Waugh (2018a), pre-tax real wages in a market with state variable $s$ equal

$$ w(s) = \omega(s)^{\theta} \mu(s)^{-\frac{1}{\theta}} z^{\frac{\theta - 1}{\theta}} C^{\frac{1}{\theta}}. \tag{31} $$

Here $\omega(s)$ is the home share defined in (30); $\mu(s) = \frac{\mu(s)}{\pi(s)}$ is the number of labor units; $z$ is domestic productivity; $C$ is aggregate consumption.

Equation (31) connects the trade exposure measure in (30) with island-level wages. A smaller home share implies that wages are lower with elasticity $\frac{1}{\theta}$. This means that if imports (relative to domestic production) are larger, then wages in that labor market are lower. Similarly, a larger home share means that wages are higher.

While this looks like the “micro-level” analog of the aggregate result of Arkolakis, Costinot, and Rodríguez-Clare (2012) it is different in one important respect: the micro-level wage response to micro-level trade exposure to trade takes the exact opposite sign.

Figure 4 illustrates these observations by plotting the logarithm of pre-tax wages by world price and home productivity so it exactly matches up with Figure 3. As equation (31) makes clear, there is a tight correspondence between wages and the home share in Figure 3. As in Figure 3, there are three regions to take note of.

The first region is where import competition is prevalent (low world prices or low home productivity) wages are low. A way to understand this result is as follows: wages reflect the value of the marginal product of labor. In import competing islands, trade results in lower prices and, hence, lower wages. The second region is where exporting is prevalent. Exporting regions are able to capture high world prices, and, thus, wages are high in these islands. Finally, the center region is where commodities are non-traded. Here, the gradient of wages very much mimics the increase in domestic productivity. In contrast, where goods are imported or exported, the wage gradient mimics the change in world prices.

Again, it is important to reflect on the stochastic nature of this economy. While the stationary equilibrium of the economy results in a stationary distribution of wages, individual islands (and households living on those islands) transit between different states (world prices and domestic productivity). For example, an island may be an exporter with households receiving high wages, but given a sequence of bad productivity shocks, the island will stop exporting, and household wages will fall.

Finally, equation (31) connects with the aggregate gains from trade. Any change in aggregate
trade exposure will also change in aggregate consumption, i.e. the $C$ term. That is all workers benefit from the “aggregate gains to trade”, but the island-level incidence will vary with its trade exposure and may mitigate or completely offset the aggregate benefits from trade.

### 4.3. Connection with Autor, Dorn, and Hanson (2013)

The preceding results relate closely to the empirical specification and evidence of Autor, Dorn, and Hanson (2013) and Acemoglu, Autor, Dorn, Hanson, and Price (2016) that link changes in trade exposure with labor market outcomes such as wages (see Section IV.B of Autor, Dorn, and Hanson (2013)). To do illustrate the connection, start with (31) and take log differences across time yielding

\[
\Delta \log w(s) = \frac{1}{\theta} \Delta \log (\omega(s)/\hat{\mu}_h(s)) + \frac{1}{\theta} \Delta \log C + \Delta \log \left( \frac{\hat{z} + 1}{\hat{z}} \right),
\]

(32)

which says that the change in wages across locations is summarized by (i) trade exposure via the change in per-worker home share, (ii) the change in aggregate consumption and (iii) the change in location-specific productivity.

Equation (32) is closely related to the empirical specification of Autor, Dorn, and Hanson (2013) (see equation (5)). Autor, Dorn, and Hanson (2013) relate various labor market outcomes at the commute zone level to commute-zone-level measures of trade exposure. Put in their terms, our theory connects changes in wages on the left hand side with trade exposure, an aggregate effect (which would be picked up by the constant/or time effect), and the error term reflects unobserved commute-zone-level productivity shocks.

Consistent with their arguments, equation (32) makes clear that an instrumental variable strategy is necessary to identify the causal effect of trade exposure on wages. Commute-zone-level productivity shocks are unobserved, but correlated with trade exposure and, thus, trade exposure could increase either because of changes in world prices or domestic productivity.

The structure of the model suggests several instrumental variable strategies. One valid instrument would be to use the world price (if observed) directly. The world price is orthogonal to domestic productivity (the exclusion restriction), yet correlated with the home trade share. The exclusion restriction follows from our small open economy assumption and the specification that the stochastic process in (14) that is assumed to be orthogonal to $\hat{z}$.

An alternative strategy would be to use another country’s imports as an instrument. Another country’s imports would be orthogonal to the home country’s productivity, but correlated with world prices. This, in fact, is quite similar to the instrument proposed in Autor, Dorn, and Hanson (2013).

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7Moveover, the model makes clear that one should be concerned, in general equilibrium, that a change in domestic productivity would feed into world prices and, thus, invalidate this strategy.
4.4. Labor Supply and Migration Responses

Figures 5 and 6 plot the policy functions for labor supply and migration in the calibrated economy. The y-axis is ordered by asset level with the origin being the borrowing limit $\bar{a}$. The x-axis is ordered twice: first by the local productivity shock $z$, and then by import exposure. Within these figures, it’s important to remember that most of the population resides near or just off the borrowing limit and in the middle of the shock distribution.

Focusing on labor supply, Figure 5 illustrates several points. First, it’s the wealth and the poor that opt out of the labor force. And these two forces become weaker as the an islands state improves. It’s a wealth effect that drives rich households to opt out of the labor force. This is a feature that our model shares with Chang and Kim (2007). The second effect—the poor dropping out—is driven by home production. Even though poor households have limited resources to survive on, home production provides them a “life line” so that when labor market conditions are poor, they may substitute out of the labor force. This latter effect is particularly acute for the poor, who are trade exposed.

Figure 6 shows migration is varying across states. First, only the wealthy migrate. Within an island, Figure 6 shows that only those with the financial means to move are able to do so. Second, within an island, the relationship is non-monotonic. Obviously, those who can not afford a move don’t. But then it gets interesting. It’s the moderately wealth who are more likely to move relative to the really wealth. The idea here is the following: migration is a substitute to smoothing consumption by drawing down one’s asset holdings. Thus, the really wealthy households are more likely to stay as they have a large enough stock of accumulated assets to smooth out there assets. The moderately wealthy do not and, hence, they opt for out migration to improve their living standards.

Across islands, those with worse productivity shock and increased trade exposure are more likely to experience out migration. Similar to the discussion above, households in the model use migration as a means to escape poor labor market conditions. This force is similar to those in Lagakos, Mobarak, and Waugh (2017).

5. Calibration

While illustrative, the previous section left open several questions. While our model will predict heterogenous responses, how the aggregate gains from trade offsets these responses it not clear. Second, our model has several margins which households can potentially partially offset reductions in wages — borrowing, labor supply adjustments, and moving. The quantitative analysis below explores these issues.

This section outlines our calibration approach. We divide our calibration approach into essen-
Figure 5: Labor Supply by Assets and Trade Exposure

Figure 6: Migration by Assets and Trade Exposure
Table 3: Predetermined Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target Moment/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor, $\beta$</td>
<td>0.95</td>
<td>—</td>
</tr>
<tr>
<td>World Interest Rate, $R$</td>
<td>1.02</td>
<td>—</td>
</tr>
<tr>
<td>Persistence of $z$ and $p_w$ process</td>
<td>0.95</td>
<td>—</td>
</tr>
</tbody>
</table>

5.1. Predetermined Parameters

**Time and Geography.** The time period is set to a year. Given the time period we set discount factor equal to 0.95. This is value is at the top end with those used in Krueger, Mitman, and Perri (2016). Geographically, in our model, there is an abstract notion of an island, households living on that island, and working within its local labor market. Per the discussion in Section 4.3, we want to tightly connect our model’s implications with empirical evidence of Autor, Dorn, and Hanson (2013). Thus, we will think of the empirical counterpart to an island as a Commuting Zone (see Tolbert and Sizer (1996)) and as used in Autor, Dorn, and Hanson (2013)).

**Productivity and World Price Process.** The productivity process in (4) and (14) leaves three parameters to be calibrated: $\{\phi, \sigma_z, \sigma_w\}$. The parameters controlling the volatility are internally calibrated and described below. The parameter controlling the persistence of the shocks is externally calibrated: we set the persistence parameter $\phi$ to 0.95. With that said, a key issue in this class of models is how persistent the shocks are and, more specifically for our question, the permanence of the change in comparative advantage. This is important in that it will affect how insurable or uninsurable these shocks are. We speculate that the results of Krishna and Senses (2014) and Hanson, Lind, and Muendler (2015) speak to these dynamics of comparative advantage, as well.

The final world price that we must calibrate is the gross real interest rate, $R$ in the initial stationary equilibrium. We set this equal to 1.02 which corresponds with a two percent annual interest rate.
5.2. Calibrated Parameters

We calibrate the remaining parameter values so that moments in the initial stationary equilibrium and moments along the transition to a new stationary equilibrium match moments about the period prior to and post China’s rise. In particular, we think of the initial stationary equilibrium period as corresponding with the first time period of 1990 to 2000 used in Autor, Dorn, and Hanson (2013); the China shock period (i.e. the transition) corresponds with the second time period or 2000-2007.

The following parameters are chosen: the disutility of work, home production, the migration cost, the borrowing constraint, pre-China shock trade cost, post-China shock trade cost, and the demand elasticity; call this parameter vector \( \Theta = \{ B, \bar{a}, m, \tau_{im}, \tau_{ex}, \sigma_z, \sigma_w, \tau_{im}', h, \theta, \sigma_v \} \). This parameter vector is then chosen to minimize the distance between eight moments in the model and eight moments in the data. Below we describe the eight moments and the parameters they are most tightly linked with.

- **Labor force participation.** We target a labor force participation rate of 67 percent which corresponds with the average value across the period of 1990-2000 in US data. We target this value in the initial stationary equilibrium. This moment is most informative about the disutility of work, \( B \).

- **Fraction of households with households have zero or negative wealth.** Krueger, Mitman, and Perri (2016) report from the Survey of Consumer Finances that approximately 40 percent of households have zero or negative wealth. We target this value in the initial stationary equilibrium. This moment is most informative about the borrowing constraint, \( \bar{a} \).

- **Migration rate.** We use the IRS migration data which uses the address and reported income on individual tax filings to track how many individuals move in or out of a county. We compute that a bit over three percent of households move across a commuting zone at a yearly frequency. We target this value in the initial stationary equilibrium. This moment is most informative about the migration cost, \( m \).

- **Trade volumes pre China’s rise.** In the initial stationary equilibrium, we target an initial import to GDP ratio of thirteen percent. This latter value is consistent with the degree of openness seen in Figure 1 in the late 1990s prior to the acceleration of Chinese trade. This moment is most informative about the initial import and export trade cost (which we assume take on the same value initially).

- **Standard deviation of growth rates in commute zone level labor earnings.** In the initial stationary equilibrium, we target standard deviation of growth rates in commute zone
level labor earnings of 6.5 percent. This value is measured in the data by using the decennial Census data from Autor, Dorn, and Hanson (2013) and focusing on the period between 1990 and 2000. One short-cut that we take is that given in the closed economy version of this model and given a $\rho$ and $\theta$, we can determine, in closed form, the volatility of growth in labor earnings simply by picking $\sigma_z$. Thus, we directly calibrate this value prior to computing the stationary equilibrium.

- **Trade volumes post China’s rise.** Along the transition path, we target an import to GDP ratio of seventeen percent seven years after the change in policy. This latter value is consistent with the degree of openness seen in Figure 1 in 2007. This moment is most informative about the final trade cost.

- **Aggregate gains from trade.** Given the long-run increase in trade from the bullet above, we can use the formula of Arkolakis, Costinot, and Rodríguez-Clare (2012) to impute the aggregate gain in output associated with a trade elasticity of four (Simonovska and Waugh (2014)). We then ensure that our model has the aggregate responses in output as the Arkolakis, Costinot, and Rodríguez-Clare (2012) would formula implies. While the Arkolakis, Costinot, and Rodríguez-Clare (2012) does formula does not hold in our model, we are using their formula as a model diagnostic to ensuring that the level of the gains from trade are the same as in a standard, representative agent trade model. This moment is most informative about the volatility of world prices, $\sigma_w$. This may seem odd, but read on. The insight here is that variation in world prices determines how elastic aggregate trade flows are to a change in trade frictions. For example, if world prices are very dispersed, then large changes in trade frictions are necessary to generate large changes in trade flows. In contrast, if there is little variation in world prices, then small changes in trade frictions will generate large changes in trade flows. This insight is analogous to the behavior of the Eaton and Kortum (2002) model where the extent of technology heterogeneity controls how elastic trade flows are to changes in trade costs.

- **Wage and labor force participation elasticity from Autor, Dorn, and Hanson (2013).** Specifically, we target the elasticities described in Table 1. These moments are most informative about $\theta$ and $w_h$. The logic about the relationship between the Autor, Dorn, and Hanson’s (2013) wage elasticity result and $\theta$ and follows from the discussion in 4, that is the elasticity of wages to changes in trade exposure is related to demand elasticity. The home production parameter $w_h$ is related to the labor force participation elasticity of Autor, Dorn, and Hanson (2013). The idea is that home production controls the opportunity cost of working in the market and, thus, it controls how households substitute in and out of the labor force in response to shocks. Given that Autor, Dorn, and Hanson (2013)
have identified the elasticity of labor supply to a trade shock, this will inform our home production parameter.

- **Migration elasticity from Greenland, Lopresti, and McHenry (2017).** We target the Greenland, Lopresti, and McHenry (2017) migration elasticity described in Table 2. This moments is most informative about the scale of the preference shock $\sigma_\nu$. For example, if the variance of the preference shocks are large, then large movements in the value of moving relative to staying are required to induce households to move. On the other hand, if the variance of the shocks are small, then small changes in the relative value of moving to staying will induce large numbers of households to move. Hence, the migration elasticity of Greenland, Lopresti, and McHenry (2017) helps pin this parameter down.

### 5.3. Implementing the China Shock

Main exercise focuses on a change in the the ability to import goods, i.e., a reduction in $\tau_{im}$. Mechanically, we implement the change in the following way: In year one, the change in trade costs is announced, but implementation is not immediate. Two periods later, the trade cost begins to linearly decrease from $\tau_{im}$ to $\tau'_{im}$ over five years. The idea here is to generate the gradual rise in trade as in the data and to mimic the narrative around the change trade policy with China’s accession to the WTO and granting of permanent normal trade relations by the United States. As mentioned above, the exact level of $\tau'_{im}$ is chosen so that after seven years from announcement, imports to GDP resemble that seen in the US in 2007.

### 5.4. The Procedure

To implement the calibration, we work through the following steps. Much of this is done in a simultaneous manner, but we describe the core steps to facilitate how we map our model into the data.

**Step 1.** Guess the parameters $\sigma_w, \theta, w_h, \sigma_\nu$.

**Step 2.** We pick the parameters $\{B, \bar{a}, m, \tau_{im}, \tau_{ex}, \sigma_z\}$ so that the initial stationary equilibrium (pre-China shock period) replicates the labor force participation rate, migration rate, net worth of households, volatility of labor earnings, and the initial volume of trade.

**Step 3.** We pick the parameter $\tau'_{im}$ and then compute the new stationary equilibrium. We compute the transition path. That is starting from the initial stationary distribution we change the trade friction and compute the transition path to the new stationary distribution. We then check that seven years after the change, the volume of trade equals seventeen percent. We compute the long run aggregate gain in output and compare it to the value predicted by the Arkolakis, Costinot, and Rodríguez-Clare (2012) formula.
### Table 4: Calibration: Parameters and Aggregate Moments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disutility of work, $B$</td>
<td>1.05</td>
<td>Aggregate participation rate</td>
<td>66</td>
<td>66</td>
</tr>
<tr>
<td>Migration Cost, $m$</td>
<td>1.75</td>
<td>CMZ. migration rate</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Borrowing Limit, $-ar{a}$</td>
<td>0.84</td>
<td>% Households with $\leq 0$ net worth</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Pre-China Trade Cost $(\tau_{ex}, \tau_{im})$</td>
<td>1.16</td>
<td>1990s Imports/GDP</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Post-China Trade Cost $(\tau'_{im})$</td>
<td>1.37</td>
<td>2007 Imports/GDP</td>
<td>16.2</td>
<td>15.4</td>
</tr>
<tr>
<td>Std. Dv. of $z$ $(\sigma_z)$</td>
<td>0.032</td>
<td>Std. Dev. in CMZ earnings</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Std. Dv. of $p_w$ $(\sigma_w)$</td>
<td>$1.64 \times \sigma_z$</td>
<td>Predicted ACR Gains</td>
<td>1.6</td>
<td>1.8</td>
</tr>
</tbody>
</table>

**Note:** All moments are reported in percent. Migration cost and borrowing limit parameters are reported as a fraction of output per worker.

**Step 4.** Given the transition path, we simulate data sets analogous to those in Autor, Dorn, and Hanson (2013) and estimate the wage, labor force participation, and migration elasticities with respect to the trade exposure metric. In particular, we constructing data analogs in our model as they are constructed in Autor, Dorn, and Hanson (2013) and estimate (2) on model generated data with a time effect. Simulated trade flows for another country (same sequence of $p_w$s, different sequence of $z$s) is the instrument.

Given the difference between the model and data elasticities, return to **Step 1.** and update the parameters $\sigma_w, \theta, w_h, \sigma_\nu$.

### 5.5. Calibration Results

Table 4 reports the calibration results parameters and the associated aggregate moments. Overall, the model is very flexible and is easily able to exactly fit most moments. The only moments in which there is some minor difficulty are those that depend upon the transition path. At the current stage, this appears to be largely due to time constraints and the solver needing more time to find the exact parameterizations.

Figure 7 plots the model’s prediction for imports. Similar to the data, the model generates rising trade exposure over the early 2000s. As we discuss below, one feature of the model is that once the path of trade costs are announced, consumption contracts by a small amount. Since demand
for all goods (including imported) are contracting, imports fall by a slight amount in the first couple of years and then slowly rise.

Table 5 reports the micro-moments, model prediction/fit, and the resulting parameter values. The first row reports the estimates from Autor, Dorn, and Hanson (2013) and Greenland, Lopresti, and McHenry (2017), the second row reports the results from the calibrated model. There are a couple of points to mention. First, our model is able to come in the close vicinity the micro-moments. All point estimates from the model lie within the 95-5 confidence bands of the data. Our interpretation of this (along with extensive sensitivity analysis) is that results of Autor, Dorn, and Hanson (2013) are informative about structural parameters that govern the underlying data generating process.

With that said, the model is experiencing difficulty in exactly nailing these moments. The difficulty appears to be in the strong substitution pattern between not working and migrating. In the model, households are unseing these two margins to avoid the negative consequences of the shock. And when one of the margins looks more or less favorable, the model implied elasticities change in different ways.

A final point of note is that the estimated demand elasticity, \( \theta \), is relatively large. The reason
Table 5: Calibration: Micro Moments and Parameters

<table>
<thead>
<tr>
<th></th>
<th>∆ Labor Earnings</th>
<th>∆ NILF</th>
<th>GLM ∆ Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>-4.30</td>
<td>1.11</td>
<td>-1.43</td>
</tr>
<tr>
<td></td>
<td>[-6.62, -2.00]</td>
<td>[0.52, 1.72]</td>
<td>[-3.33, 0.48]</td>
</tr>
<tr>
<td>Model</td>
<td>-4.10</td>
<td>1.24</td>
<td>-1.92</td>
</tr>
</tbody>
</table>

Demand elasticity $\theta$  
Home production $w_h$  
Moving shock variance $\sigma_{\nu}$

Parameter Values  
9.53  
0.22  
0.96

Note: Values in brackets report 95-5 confidence intervals.

follows from the discussion in Section 4: The elasticity of wages to changes in trade exposure is related to one over the demand elasticity. So if the demand elasticity is high, then changes in trade exposure will weakly pass-through into wages. Hence, what this is telling us vis-a-vis the Autor, Dorn, and Hanson (2013) evidence, is that pass-through was not that strong.

6. The China Trade Shock

This section analyzes the affects of a change in the ability to import goods on the economy at the macro and micro level.

6.1. Aggregate Outcomes

The top panel of Figure 8 illustrates the behavior of aggregate consumption, labor supply, and output both before and after the trade shock.

In response to the trade shock, aggregate consumption is essentially unchanged for the first several periods and eventually increases by one percent after seven years. Why does aggregate consumption does not increase more strongly? One might speculate that, in net present value terms, agents are richer because of the gains from trade and, thus, they should increasing consumption today to enjoy future wealth. The reason this does not occur is because of incomplete markets and precautionary savings motives. A reduction in trade costs increases the amount of uninsurable income risk in the model; this mechanism is consistent with the findings of Krishna and Senses (2014). The increase in income risk increases precautionary savings motives (see, e.g., Huggett and Ospina (2001) ) and, in turn, leads to an increase in aggregate savings and an improvement in the trade deficit.
The blue dashed line reports the change in labor supply. Aggregate labor supply increases, but then declines to a little less than one percent above its pre-trade shock level. Like the forces driving consumption dynamics, precautionary motives are why aggregate labor supply increases. Working more today yields additional savings to guard against the increase in uninsurable income risk.

An important point to take note of that these aggregate increases in labor supply are consistent with Autor, Dorn, and Hanson (2013) finding that increases in trade exposure lead to reductions in labor force participation. By construction, our model is matching their evidence. However, as mentioned before, this evidence has little content when speaking about aggregate effects. In this case in particular, the aggregate response is in the exact opposite direction of a naive interpretation of Autor, Dorn, and Hanson (2013) labor force participation evidence.

Increases in aggregate labor supply lead to increases in aggregate output—this is the black dotted line in Figure 8. Initially the increase in output mimics one for one with the increase in labor supply. However, once trade begins to expand in 2003, the gains from trade begin to kick in as output expands above and beyond the change in labor supply.

While the trade shock increases output, it is not passing through to aggregate consumption. This implies that the trade deficit is decreasing as the trade shock hits. That is, a reduction in the cost of importing, lead exports to expand by more than imports. This suggests that attributing an expansion of the US trade deficit in the early 2000s to increased trade with China is wrong and suggests the global financial forces advocated by Bernanke (2005) are a more likely culprit.

The improvement in the trade deficit and the consumption response are intimately related. There are two ways to see this. A traditional open economy macro perspective is to view this result through the identity that output minus consumption (aggregate savings) equals trade deficit. In response to the trade shock, output is increasing, consumption is not, thus the trade deficit must improve. And the reason here is about precautionary savings motives, households work more and save more as there is an increase in uninsurable income risk.

It is also consistent with a traditional international trade perspective, i.e., that the trade deficit equals exports minus imports. First, a muted consumption response implies that demand for all goods (foreign and those domestically produced) is relatively unchanged. Given production is increasing, exports must increase as product not consumed domestically goes abroad. Moreover, imports do not increase since aggregate demand is unchanged. Thus, exports increase, imports remain unchanged, and the trade deficit improves.

6.2. Welfare

How does this all add up in terms of welfare? We compute welfare gains or loses as lifetime consumption equivalents, including the transition. That is the permanent increase in consump-
Figure 8: Model: Consumption, Labor Supply, and Trade Imbalance
tion that a household must receive to being made indifferent between living in the world prior to the shock and after.

Table 6: Welfare

<table>
<thead>
<tr>
<th></th>
<th>Welfare (Baseline)</th>
<th>Welfare (Low $\theta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>0.89</td>
<td>0.82</td>
</tr>
<tr>
<td>Import Exposed</td>
<td>−0.14</td>
<td>−0.30</td>
</tr>
<tr>
<td>[ 0.09 ]</td>
<td>[ 0.27 ]</td>
<td></td>
</tr>
<tr>
<td>Non-Traded</td>
<td>0.75</td>
<td>0.55</td>
</tr>
<tr>
<td>[ 0.64 ]</td>
<td>[ 0.25 ]</td>
<td></td>
</tr>
<tr>
<td>Export Exposed</td>
<td>1.70</td>
<td>1.52</td>
</tr>
<tr>
<td>[ 0.27 ]</td>
<td>[ 0.47 ]</td>
<td></td>
</tr>
</tbody>
</table>

Note: Values are lifetime consumption equivalents; values in brackets report the share of the population in that category.

Table 6 reports the welfare effects from the shock. Figure ?? provides a visualization of the welfare gains by island type and their trade exposure. The average welfare gains is reported in the first row. This is computed by integrating individual value functions across asset holdings and states and computing the consumption equivalent for the “average” individual. We find these are a bit less than one percentage point in consumption equivalents. As discussed above, these gains—by construction— are consistent with Arkolakis, Costinot, and Rodríguez-Clare (2012). There a many reason why the average gain will be lower then the long run increase in output per worker. One key reason is the departure from the representative agent assumption and averaging over.

As expected, the average value hides a lot of heterogeneity. The last three rows of Table 6 report the gains associated with initial condition of trade exposure and in brackets the fraction of the population that is classified as such. Eight percent of the population was initially import exposed and they experienced modest welfare losses, almost −0.14 percent. In contrast, twenty percent of the population was initially export exposed and they experienced large welfare gains on the order of almost two percent.

What role does the Autor, Dorn, and Hanson (2013) evidence play in shaping these welfare measures? To illustrate this point, we performed the following exercise. We re-calibrated the model as if the wage elasticity in Table 1 was twice as large. That is rather than being −4.3, we
Welfare: Trade Shock

- Average Welfare
- Import Exposed
- Non-Traded
- Export Exposed

Welfare: Low Theta

- Average Welfare
- Import Exposed
- Non-Traded
- Export Exposed
targeted a value of -8.0 while keeping all other moments fixed (aggregate and micro). Consistent with our interpretation of the parameter estimates, the key value that changes in this alternative calibration is the parameter $\theta$ declining from our baseline estimate of 9.5 to 4.7.

The second column of Table 6 and 6.2. There are a couple of points to note. First, the average is about the same. However, the losses are larger in two respects. Conditional on losing the welfare losses are larger. And more of the population actually experiences a loss in welfare. Similarly, the winners from trade win slightly less than in the baseline parameterization, but overall there are more winners. In other words, the Autor, Dorn, and Hanson (2013) evidence is informative about the cross-sectional dispersion in welfare gains and losses. How meaningful it is for aggregate gains from trade and aggregate responses is still not clear.

7. Trade and Financial Shocks

The second shock that we entertain is a “global savings glut” type shock (see, e.g., Bernanke (2005)) that reduces the world real interest rate $R$. This will change the ability/desire of households to trade intertemporal. In year one, we lower the real interest rate so that the $R - 1$ decreases by 200 basis points permanently. Immediately during the rapid rise in Chinese trade during the early 2000s, short-term real interest rates fell by 400 basis points. To be conservative, we change the real interest rate by only half this amount.

8. Conclusion

our model abstracted from many things, especially non-traded factors within a local-labor market—housing is one of these. Computationally, the introduction of housing is challenging. However, the economic environment of the United States in the 2000s’ were basically characterized by two events: a housing boom and bust and an unprecedented explosion in import exposure to Chinese trade. For example, the evidence of housing masking of the decline in manufacturing in Charles, Hurst, and Notowidigdo (2016) raises many questions about how these two events relate.

Lots more work todo!
References


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—— (2018b): “Redistributing the Gains From Trade Through Progressive Taxation,”.


A. Connection with National Accounts

This section connects these equilibrium relationships to national income and product accounts (NIPA). This will help facilitate an understanding of the connection between trade imbalances and household’s consumption-savings decisions. Note that in all of this derivation, we normalize the price of the final good to one.

The Income Side of NIPA. It is first useful to start from an income side measure of production (or GDP) in our economy. Given competition, the value of aggregate production of the final good must equal aggregate payments for intermediate goods. The latter equals aggregate payments to labor in the production of all intermediate goods.

\[ Y = \int_s w(s) \mu(s) \]  

Examining (??) and (33) allows us to connect aggregate income with consumption. Specifically, by integrating over the consumers budget constraint (ignoring home production), then noting how total value added must equal wages, one can substitute (33) into (??), then we arrive at the following:

\[ Y = C - RA + A' + \int_a \int_s m_{m}(s,a) \lambda(s,a) \]  

so aggregate income equals consumption (private and public) minus (i) returns on assets (ii) new purchases of assets (iii) plus moving costs.

This basically says that income/production must equal consumption net of income not associated with production (i.e. returns on assets) plus “investment” in assets and moving costs. For example, if consumption is larger than income one reason is that (in aggregate) households (on net) are borrowing from abroad (\( A' < 0 \)).

Production Side of NIPA. The the value of aggregate production of the final good must equal the value of intermediate goods production

\[ Y = \int_s p(s) z \mu(s) \]  

which we can then connect with the expenditure side of GDP through the market clearing conditions for intermediate goods and final goods. Specifically, by connecting the production side with the demand side for non-traded goods in (19), imports in (21) and exports in (23) and
the equating final demand with consumption we have

\[ Y = C + \int_s p(s)\text{exports}(s) - \int_s p(s)\text{imports}(s). \]  

(36)

Or GDP equals consumption (market) plus exports minus imports.

**Savings, Trade Imbalances, and Capital Flows.** Finally, we can connect the income side and the production side the national accounts to arrive at a relationship between asset holdings and trade imbalances. By working with both (34) and (36) we get the following relationship

\[ Y - C = \int_s p(s)\text{exports}(s) - \int_s p(s)\text{imports}(s), \]  

(37)

\[ = - rA + (A' - A) + \int_a \int_s m(s, a)\lambda(s, a), \]

where \( r \) is the net real interest rate. This relationship says the following: aggregate savings equals the trade imbalance. And this, in turn, we can connect the trade balance with the savings decisions of the households. That is the trade balance equals payments on net asset holdings plus net change in asset holdings (adjusted for moving costs). To map this into Balance of Payments language: the trade imbalance plus foreign income payments is the current account; the capital account is the net change in foreign asset holdings; then (we suspect) moving costs would show up as the “balancing item.”

To see this, consider the special case where moving costs are zero, no home production, and tariffs are zero. Then we have the relationship

\[ Y - C = \int_s p(s)\text{exports}(s) - \int_s p(s)\text{imports}(s) = - rA + (A' - A). \]  

(38)

Here if exports are greater than imports, then this implies that the households are doing several things. The trade surplus may reflect that households (on net) are making debt payments (\( rA \) is negative). Second, the trade surplus may reflect that the households (on net) are acquiring foreign assets (\( A' - A \) is positive). Finally, note that in a stationary equilibrium, the trade imbalance only reflects payments from foreign asset holdings. This implies that the current account and capital account are always zero in a stationary equilibrium, but that trade may be imbalanced.
B. Computational Appendix

This section describes our computational approach. Related materials are posted at https://github.com/mwaugh0328/redistributing_gains_from_trade which retains code for our companion paper in Lyon and Waugh (2018b). The code is presented in two different languages: Matlab and Julia. The implementation in both languages follows the same core steps, but the details are slightly different. The discussion below follows the Matlab code. For a detailed explanation of how the Julia code works, see the file julia/README_Julia.md.

2.1. Computing and Solving the Stationary Equilibrium

Below we describe how to compute and solve the stationary equilibrium of the model.

- We approximate the continuous asset, productivity, and world price states by discretization. The asset space follows a non-uniform grid with grid points clustered near the borrowing constraint. The number of grid points was set to 50; the results are not sensitive to increases in this number. We use the method of Rouwenhorst to discretize the productivity and world price process. We use 10 states for productivity and world prices each, thus there are 100 different states $s$.

- Guess a proposed price function $\hat{p}(s)$.

- Compute wages.

- Solve the households problem in (9). This is performed using value function iteration. One technique we use to facilitate finding a solution to the equilibrium is to “smooth” out the discrete choice problem. We do this by assuming that there are additive logistically distributed preference shocks with parameter $b_{\text{smth}}$ and these preference shocks are independently distributed across each choice. These enter into the choice problem in (9) by adding onto each option. What this gives rise to is, for each asset holdings state and state $s$, there will be a non-zero mass of households choosing all options. The probabilities take the familiar logit form. We tune the parameter $b_{\text{smth}}$ to ensure that it is small and not affecting the economics of the problem, but at the same time ensure that we find a solution.

Smoothing in this manner is important as it facilitates the use derivative based solvers in finding an equilibrium $p(s)$. This in turn results in a dramatic speed-up in the computation of an equilibrium. See, e.g., Morten (2016) who employs a similar approach.

- Given the policy functions associated with the solution to the problem in (9), we compute the stationary distribution over assets and states $s$, i.e. $\lambda(a, s)$ which is of size 50 (for each
asset state) and 100 for each state $s$. This process is sped up using sparse matrices, see the code `island_invariant.m` for details.

- Given the stationary distribution, we can compute excess demand functions for all islands $s$ which also must respect the inequalities implied by (20) and (22). These conditions imply that the problem of finding a price function consistent with a stationary equilibrium can be represented as a mixed complementarity problem (see, e.g., Miranda and Fackler (2004)). To smooth out the nondifferentiability issues with the complementarity problem we pass the excess demand functions through Fischer’s function. Again, see Miranda and Fackler (2004).

- Update $\hat{p}(s)$ and proceed until convergence criteria are met.

In solving for the equilibrium, we employed derivative based solvers. One solver that we found much success with is the `c05qc` solver from Numerical Algorithms Group. MATLAB’s `fsolve` with central finite differences performed well too.

Part of our calibration approach employs the following technique. Let $\Theta$ be the parameter vector we chose to match some moments in, say the initial stationary equilibrium. We then jointly solved for $\{p(s), \Theta\}$ in one step. That is we asked the algorithm described above to find a price vector and set of parameters such that (i) equilibrium conditions are satisfied and (ii) model implied moments match our target empirical moments. This avoided a more standard, but time consuming approach of guessing a $\Theta$, solving for an equilibrium, updating $\Theta$, etc. Extensive sensitivity analysis found no issues surrounding multiplicity of equilibrium.

2.2. Computing the Transition Path

To be completed soon.